Developing a Cross-institutional, Socio-cultural Research Agenda Leading to Rich and Equitable Mathematics Education

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This paper describes the ten-year evolution of a research agenda that has resulted in improved mathematics education and progress toward closing the achievement gaps among demographically diverse student groups. The research is based on work with whole school districts as units of analysis and facilitating change through collaboration between mathematicians, mathematics educators and public school personnel including teachers and administrators. New methodologies, and the development of strong inter-disciplinary and cross institutional research partnerships, are described in terms of research designs which can link theory and practice and result in building the capacity of school districts to improve mathematics leadership, teaching and learning.

Robert Moses, a former civil rights leader and current facilitator for the Algebra Project (Moses & Cobb, 2001), was one of the first people to make the case that access to rich mathematics learning is indeed a new civil right. Students who are not able to pass mathematics and science classes are truly limited in life options in today’s highly technological and scientific world. Numerous reports (Schott Foundation for Public Education, 2009; New Mexico First, 2009; National Mathematics Advisory Council, 2008; National Academy of Sciences, National Academy of Engineering, & Institute of Medicine, 2007), as well as national (NAEP, 2009) and state test data (Brandi, Freehill, & Frampton, Eds., 2009) continue to show a large achievement gap between linguistically and ethnically diverse students and mainstream students in mathematics and science. According to these reports, the problem is rooted in the fact that the fastest growing demographic groups of diverse students do not have adequate preparation in mathematics and science to be able to complete required advanced courses and qualify for 21st century careers.

Theoretical Framework
This paper describes the development of a research agenda in mathematics education over the last ten years by a collaborative group of mathematicians, math educators, researchers and public school and state leaders that has resulted in significant achievement gains for students, teachers, and schools who are part of non-dominant communities. The current collaborative research team
began a focused effort in the 1990’s to address the mathematics learning needs of minority-majority schools and students in low-income Hispanic/Latino communities in New Mexico. In addition, New Mexico has widely-separated rural communities, a high rate of poverty and a large number of students who come to school speaking a language at home other than English. In many ways, the state of New Mexico is an educational lab for the future of the United States which will soon reflect the demographics of this minority-majority state.

From the beginning, the research team believed that the problem of poor performance in mathematics should not be blamed on the student. The research team rejected the deficit-view of education (Trent, Artiles, & Englert, 1998) which suggests that a child’s educational success is determined largely by his or her ethnicity, family income, or language. The team also believed that weak mathematics achievement could not be alleviated by isolated teacher professional development (PD) that doesn’t consider the context or culture of the district. As part of the theoretical grounding for enactment of mathematics reform, the research team valued explicit, collaborative organizational work between practitioners in school settings, research mathematicians, mathematics educators and educational researchers in university settings (Wiburg & Lozano, 2001).

Our research agenda in mathematics education has emphasized the use of data from schools and the use of research findings to guide decisions about next steps and actions. The use of design-based research with frequent feedback to partners has evolved during the growth of our agenda. In addition, our evolving research agenda reflects a growing recognition that productive partnerships require understanding of the systems in which a school district operates, both the external community in which the district is situated and the internal district organization. We have adopted an evolving definition of systems thinking as it relates to managing change as part of improving mathematics education.

Very simply, a system is a collection of parts (or subsystems) integrated to accomplish an overall goal. If one part of the system is removed, the nature of the system is changed (McNamara, 2007). Complex systems, such as school district organizations, are comprised of numerous subsystems, as well. Each subsystem has its own boundaries, and includes various inputs, processes, outputs and outcomes geared to accomplish an overall goal for the subsystem.
Systems thinking provides a new perspective for leaders who are learning to look beyond individual events to interpret the patterns and structures of the system and subsystems in terms of the desired outputs of the whole system.

An understanding of the system includes recognizing the political forces at work in a district and the extent to which all students have access to equitable opportunities to learn. The frequently discussed achievement gap in mathematics may have more to do with the system of access to rich mathematics learning than to any individual characteristics of students. Trying to fix the problem with only individualized strategies, rather than system-wide approaches in which all students are expected to learn and achieve, limits the implementation of mathematics reforms.

The history of education is replete with examples of pendulum swings between opposing educational approaches, e.g., whole language and phonics, or basic math skills and problem solving in mathematics. These approaches are often offered as “magic bullets” to districts without knowledge of how an approach needs to be integrated into the whole system and made sustainable. Our own journey in mathematics education began with isolated approaches to professional development such as inviting teachers to summer academies and then seeing very little change in classrooms. Since 2007, participation in our university/public school partnership requires a commitment from the whole district, which includes both a financial investment and structural change, such as providing teachers with time, materials and mentoring as they are asked to teach in unfamiliar ways.

This paper provides an in-depth look at several innovative research projects that take a systems view of problems related to poor performance in mathematics by many under-represented students. Information about these projects includes the kinds of research methodologies and university-public school collaborative work that we believe are essential for using research to contribute to the public good, in this case the preparation of all students to succeed in future careers.

The Research Projects
The representation below shows the interconnectivity of the various grant programs discussed in this article and how the learning from one program informed future grants.
In many ways the research agenda that has been developed has its foundations in a university-district project which was one of the last National Science Foundation (NSF) Teacher Enhancement Grants, the Gadsden Mathematics Initiative.

**The Gadsden Math Initiative (GMI)**

In May of 2000, the Gadsden Independent School District (GISD) served approximately 14,000 students in fourteen elementary schools, three middle schools and three high schools. The district consisted of many small rural schools in a large geographic area along the Rio Grande. Over 90% of the students were Hispanic and more than 60% came from homes where English was not the first language. The average family income was below $20,000 per year and one hundred
percent of the students in the district were eligible for free lunch. All K-8 campuses were
designated as Title I.

The turnover rate of teachers in the district was very high—almost 100% every four years. There
was no district-wide math curriculum in 2000 and each school taught mathematics in any way it
desired. The professional development of teachers was sporadic and not necessarily related to
what teachers were doing in their classrooms.

Academic proficiency in GISD was dismal in 2000, and the district was one of the lowest
achieving school districts in the state of New Mexico. It ranked 88 out of 89 districts in terms of
mathematics achievement scores on the state-mandated assessment, the New Mexico Standards
Based Assessment (NMSBA).

During the five-year GMI, principals and teachers across the district were regularly provided
with high quality professional development, all K-8 schools were implementing a rigorous
standards-based mathematics curriculum, teachers were provided classroom-based support by
one of twenty district math coaches, and new teachers were required to participate in a year-long
new-teacher induction program for mathematics. Most importantly, the academic proficiency of
students rose and the achievement gap between Gadsden students and the rest of the state began
to narrow.

**Improvements in Mathematic Achievement**
The results of the GMI can be summarized by examining the effectiveness of the initiative in
improving the mathematics learning of the students. In terms of math achievement, GISD moved
from 88th out of 89 districts in 2000 to 32nd out of 89 districts in 2007. Graph 1 below shows the
percent of proficient and advanced students in GISD on the mathematics NMSBA compared to
the state average in 2000, the beginning of the GMI. This graph is followed by Graph 2 which
details the 2007 NMSBA results at the end of the GMI. The GISD scores continued to rise even
after funding ended, and at some grade levels the GISD percentages continue to surpass those of
the state.
As shown below in Graph 3, gains in mathematics achievement continued in most grade levels even after the GMI ended. The chart displays the NMSBA data from 2000 (the beginning of GMI), 2007 (the end of GMI), and 2009 (two years after the end of GMI). Through the GMI partnership, the Gadsden Independent School district has built the capacity to sustain and enrich mathematics learning for all of its students. Moreover, there was a significant decrease in the variability of student scores on the math tests and thus a raising of all student achievement scores. More of the students were getting quality instruction and the outcomes for students were higher achievement and a lessening of the achievement gap.
Of special note is how much higher GISD proficiency rates for English Language Learners (ELLs) were in 2009 compared to the state. In fact, the GISD proficiency for ELL students surpassed the state average in all grade levels 3-8. (See Table 1 below.)

**Table 1: Comparison of the Mathematics Proficiency Rates on the 2009 NMSBA for ELL Students in Gadsden ISD and for ELL Students in the State**

<table>
<thead>
<tr>
<th>ELL Student Proficiency</th>
<th>Grade 3</th>
<th>Grade 4</th>
<th>Grade 5</th>
<th>Grade 6</th>
<th>Grade 7</th>
<th>Grade 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>District Average</td>
<td>76%</td>
<td>66%</td>
<td>59%</td>
<td>56%</td>
<td>45%</td>
<td>66%</td>
</tr>
<tr>
<td>New Mexico Average</td>
<td>47%</td>
<td>63%</td>
<td>48%</td>
<td>48%</td>
<td>36%</td>
<td>34%</td>
</tr>
</tbody>
</table>

Development of a Systems-Based Building Capacity Model

At the same time that we were studying student achievement and its possible relationship to teacher professional development and classroom environments, we wanted to study the district as a system that had evolved by year 3 into a successful top-down and bottom-up systemic effort to improve mathematics achievement for all students. Table 2 below, an input-process-output table, describes what happened during the GMI in terms of alignment of the curriculum, teaching quality and administrative and community support. The outputs in this table suggest the elements of a systems approach that can be used by other districts to build capacity for achievement in mathematics. This model was refined during the GMI through extensive data feedback loops based on: 1) district evaluation; 2) feedback from district teachers, staff and administrators;
3) district-hired external evaluators in year 3 of the program; 4) NSF-required random classroom observations by external, trained Horizon evaluators each year; and, 5) feedback from the Student Outcomes Study was also provided to the district (Wiburg et al., 2007). While we did not have the staff to do comprehensive design-based research, we did have a very strong partnership with the district built on many years of collaborative work and an earlier technology grant and there were continuous adjustments to the project leading to an emerging model for district-wide success by the end of the GMI funding.

Table 2: Changes in Mathematics Education during the Gadsden Math Initiative

<table>
<thead>
<tr>
<th>INPUT: Problems Prior to Systemic Reform</th>
<th>PROCESS: Successive Changes in the System</th>
<th>OUTPUT: Critical Elements of the Successful System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quality Aligned Curriculum</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Each school used a different mathematics curriculum.</td>
<td>• Year 1 and 2 focused on alignment of math standards, mathematics teaching and assessment of learning.</td>
<td>• There are high expectations for all students.</td>
</tr>
<tr>
<td>• Results of student learning assessments were not used to guide instruction.</td>
<td>• Years 3-5 PD moved to site-based work with more emphasis on formative assessment, and teachers’ learning what students know.</td>
<td>• There are high levels of student engagement including generative learning and problem solving.</td>
</tr>
<tr>
<td>• There were low expectations for students, especially Spanish-speaking students (deficit thinking).</td>
<td>• A rich, rigorous curriculum was implemented with high expectations for students to think and write math.</td>
<td>• There is extensive math discourse in the classroom and teachers building on students’ multiple approaches.</td>
</tr>
<tr>
<td>• There was low achievement especially for students in SPED and ELL.</td>
<td>• Instructional strategies were integrated and expanded for ELL (K-8) and SPED (K-6) students.</td>
<td>• 2007 achievement scores for ELL students in K-8 are equal or above the state average.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>INPUT: Problems Prior to Systemic Reform</th>
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<th>OUTPUT: Critical Elements of the Successful System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher Quality and Collaboration</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• There was high teacher turnover, as much as 100% every four years.</td>
<td>• Extensive professional development is provided for all teachers including a teacher induction program for all new teachers.</td>
<td>• Teacher turnover reduced significantly in the district.</td>
</tr>
<tr>
<td>• Teachers worked as individuals, not as a professional team.</td>
<td></td>
<td>• There is extensive professional development for teachers including Lesson Study.</td>
</tr>
</tbody>
</table>
There was no consistent time requirement for teacher collaboration.
There is little consistent district-wide professional development.
There is little time spent understanding standards or assessments.
Teachers had low math skills and needed the textbook.
Teaching was viewed as telling students what to do.

Unit or lesson study is used to help teachers work together to address student needs.
Math Specialists provided monthly school based PD in years 1-2. Math Coaches hired at school sites in year 3.
Teachers learned more math through professional development.

Dedicated professional learning community time during the school day is set aside—1 hr/week.
Site-based mathematics coaching by instructional specialists is in place.
Principals monitor instructional coaching.
Teachers are more confident with math and teach by facilitating conversations rather than lecturing.

<table>
<thead>
<tr>
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<th>PROCESS: Successive Changes in the System</th>
<th>OUTPUT: Critical Elements of the Successful System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Administrative Support</td>
<td></td>
<td></td>
</tr>
<tr>
<td>There was a weak central administrative structure—each principal is doing his/her own thing.</td>
<td>Principals were required to attend 40 hours of professional development.</td>
<td>All administrators are supporting teachers in collaborating to provide a high quality mathematics curriculum accessible to all students.</td>
</tr>
<tr>
<td>Little professional development was offered to administrators in instruction, assessment, and mathematics.</td>
<td>There existed a top-down support for redesign of math teaching and learning.</td>
<td>All administrators are monitoring time and quality of instruction, as well as student achievement on short cycle and long-term assessments.</td>
</tr>
<tr>
<td>Principals weren’t monitoring instruction and teacher collaboration.</td>
<td>Time was required and processes developed for all teachers to teach math every day.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Quality of instruction was monitored by principals, central administrators and math coaches (Math Process Trainers).</td>
<td></td>
</tr>
</tbody>
</table>

As a result of the data gathered and the studied impact of the changes that the GMI had on how mathematics was taught and learned in GISD, by the end of the GMI there was a final systems design that we believed could help districts to build capacity for mathematics achievement for all students. The design included: aligning district practices and procedures to ensure full participation in and sustainability of the professional development; implementing a curriculum aligned with state and district standards in every classroom; developing and instituting campus level support for teachers through math coaches; and engaging administrators in the process of school reform.
What We Have Learned from the GMI

The GMI gave us the opportunity to learn a great deal about the complexity and value of partnerships between universities and school districts. We learned that a well defined system with common goals and expectations will facilitate positive change in mathematics teaching and learning. In order to attain higher and sustainable student achievement in mathematics, several key elements emerged:

- Strong university/district partnerships with explicit, agreed-upon beliefs and goals;
- Strong district leadership willing to make tough decisions for the betterment of student learning;
- Enactment and monitoring of a well-defined, problem-solving-based mathematics curriculum;
- Intense, systematic professional development for all stakeholders including teachers, principals, and central office administrators;
- Regular, intentional teacher collaboration to not only monitor student achievement, but to also discuss and implement specific actions to improve student learning.

We also learned about the importance of accountability for all involved parties and the value of common assessments being utilized across the district. We learned that many teachers in the district did not have strong backgrounds in mathematics, and the profession development providers were not always able to provide the depth of mathematical thinking needed to impact classroom practice. This was the beginning of our current collaboration with mathematicians from the university mathematics department. Previously the mathematicians had worked with a local school district to introduce open-ended problems into high school mathematics classes and fortunately continued to be open to working with partner districts in mathematics. Just as the educators providing PD needed the university mathematicians for their content expertise, the mathematicians learned that in order to be most effective in bringing about change in mathematics instruction, they needed educators who understood the school system. This close relationship between mathematicians and educators has been one of the defining components of our evolving mathematics research agenda.
After the GMI

Two additional large mathematics education initiatives were funded in 2004 and 2007. Both build on the *Gadsden Mathematics Initiative*. The first grant-funded program is currently in its seventh year of funding from the U.S. Department of Education’s Math Science Partnership Program through the Public Education Department in New Mexico. Close collaboration between mathematicians and mathematics educators is central to this project and includes providing professional development through summer academies and school-based follow-up mathematics coaching. A second initiative is a research program to investigate the replicability of the GMI systems model and its success in a larger, more ethnically diverse district.

**Mathematically Connected Communities (MC²)**

The mission of *Mathematically Connected Communities* is to improve student achievement and the teaching and learning of middle and high school mathematics by building a statewide learning community of mathematics educators, mathematicians, and public school leaders. Since 2004, MC² has collaborated with school districts and other educational entities throughout New Mexico to deliver mathematics professional development to middle school teachers and administrators. MC² provides three major types of PD in mathematics to whole districts for:

1) individual teachers at the classroom level to help establish and maintain an inquiry-based learning environment, and to assist teachers in strengthening their math content knowledge;
2) school administrators to help explore and develop strong mathematics leadership; and
3) professional learning communities (PLCs) to ensure that teacher collaboration is intentional, effective, and based on student data and achievement.

Each year MC² staff work together with university mathematicians and other partners around the state to organize one-week MC² Summer Mathematics Academies. These academies are held four times during the summer in different parts of the state to accommodate as many educators as possible. Table 3 below shows the number of participants, number of districts, and number of schools represented at the mathematics academies for the last four years.
### Table 3: MC² Summer Academies

<table>
<thead>
<tr>
<th></th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
</tr>
</thead>
<tbody>
<tr>
<td># of Participants</td>
<td>283</td>
<td>242</td>
<td>218</td>
<td>298</td>
</tr>
<tr>
<td># of Districts Represented</td>
<td>24</td>
<td>24</td>
<td>28</td>
<td>25</td>
</tr>
<tr>
<td># of Schools Represented</td>
<td>81</td>
<td>73</td>
<td>81</td>
<td>90</td>
</tr>
</tbody>
</table>

More than 50% of the 2010 participants had attended at least one of the academies in prior years.

A second component of MC² is the support provided to teachers during the academic year. In 2009-2010, MC² field specialists facilitated over 2,500 hours of PD to over 400 teachers from 81 schools in New Mexico.

MC² has evolved over the years. This is one of its strengths. The MC² team is constantly examining their practice and its effects (both intended and unintended) on the teaching and learning of mathematics. The university team, made up of mathematicians, mathematics educators and educational researchers have also provided feedback on data findings to districts and helped them find ways to address student learning needs. In addition, MC² developed an advisory group made up of superintendents and central office administrators and curriculum directors to provide feedback to MC². Some changes over the years have been major, while others involved “fine-tuning” or “tweaking” of procedures.

### MC²—The Journey

In late 2003, staff from the GMI approached the Department of Mathematical Sciences at NMSU to request support to continue the work stemming from the GMI. Most notably, they were looking for people with an interest in grounding the pedagogical work they had been doing with teachers in deeper mathematics. They found a research mathematics department with leadership interested in mathematics education. The current and previous department heads had been involved in funded programs on university calculus (NSF Grant #USE88-13904), and high school teacher enhancement (NSF #TPE90-53580). These programs incorporated the idea of learning mathematics by solving deep, open-ended problems. The math educators had learned the importance of problem solving for learning mathematics which had worked well in the GMI, so the intellectual fit was immediate. Thus began a partnership of mathematics educators and research mathematicians with a desire to impact public mathematics education. The core ideas for this PD were to design and present summer academies on mathematics and pedagogy, then
follow up this work by visiting participating teachers throughout the school year. However, the first three years of MC² suffered from errors in delivery and design.

Mathematicians and educators were not prepared to work collaboratively when MC² began in 2004. While they had a philosophical agreement about how mathematics should be learned, the mathematicians worked mostly with other mathematicians and educators worked mostly with other educators. In spite of positive intentions, the mathematics instruction and the education instruction were designed and delivered independently at the summer academies, and the mathematicians were not involved in the school-based follow-up except as advisors to the field specialists.

Another design issue they faced in the first phase of MC² was that initially, like many teacher professional development projects of the time, the team worked with individual teachers. For the first round of funding (2004-2007), they proposed to work with individual teachers based on their interests. During that period, many issues were identified that inhibited the impact of this work. For example, students would gain little to no positive impact from the work with teachers if what the teachers learned was not implemented in the classroom. Another finding was that many teachers found it difficult to implement changed practices without support from principals, many of whom did not understand the need to change mathematics teaching and learning. These issues predicated changes made to the program and incorporated in the second round of state funding (2007-2010).

As a result of understanding some of the barriers to change in an individual teacher project, two major changes were made during the second phase of MC². These changes included a decision to insist on district-wide participation and to provide administrator training. In this phase, professional development was provided only to teachers in districts that formally partnered with MC² by signing a memorandum of understanding and agreeing to provide 1) resources to support the teachers and 2) data so the effect of the math PD on student achievement could be studied. Further, district and school administrators were required to participate in specially-designed academies to help them understand inquiry-based mathematics instruction and to become instructional leaders for mathematics in their schools and districts.
There were other big changes that occurred at this stage. Mathematicians and educators had learned how to talk and work together, so they were better able to design and deliver much of the PD together. These content experts and educators after 2007 sought opportunities to co-present to teachers and to have educators present some of the math and mathematicians present some of the pedagogy. This transition from working as separate specialists from different disciplines into a more cohesive group that blended input from both communities greatly enhanced the value of the PD provided to teachers.

By this point, the mathematicians understood the idea of grounding PD for teachers in the school-level material that they taught. Thus, the goals of MC² changed from preparing interesting mathematics for teachers to study, to one of preparing deep mathematics that began with a look at the curriculum materials the teachers were using. In addition an effort to vertically align mathematics across the grade levels began in the summer of 2009. The MC² Summer Math Academies introduced the study of vertical trajectories of mathematical topics. One such trajectory was the development of geometry concepts across grade levels—from simple measurement concepts in second grade, through the Pythagorean Theorem in middle school, to the distance formula in high school. These sessions engaged teachers from many different grade levels in conversations about how the content they teach is related to the content in other grade levels. The leaders of MC² believe that if teachers understand the mathematical concepts students should learn before they come to their classroom, and after they leave their classroom, they can work together to ensure that students get the conceptual foundation they need to succeed in later mathematics classes.

Around the fifth year of the MC² project, the MC² staff began to invite selected teachers to help design the content of the summer academies and to present that content in the summer. The belief was that such an arrangement would help ground the summer work in the day-to-day lives of teachers and give the facilitating teams a representative from the classroom. As a bonus, these selected teachers received mentoring and leadership training.

In 2010, as the second round of funding was ending, the group responded to an RFP from the state with a proposal to continue the MC² work and unite the three state-funded mathematics professional development programs that had existed for the past six years. The focus of the third phase of MC² is threefold: 1) continue to provide PD to in-service teachers around New Mexico
based on studying the vertical learning trajectory of several key pieces of mathematics that permeate the school curriculum; 2) provide multiple levels of support to participating districts; and, 3) create a state-wide collaboration of mathematics PD providers in New Mexico.

In the latest round of MC², a strand dedicated to mathematics for English Language Learners was also added to the summer academies lead by an instructor with a strong background in mathematics and bilingual education. Success with ELL students continued to vary between districts. English Language Learners in Gadsden continued to do well but in some districts, especially when ELL students were a distinct minority, there continued to be a large gap between them and the white students in the district. Follow-up study is necessary to see if this new strand in the academies will help districts provide better mathematics service to ELL students.

As MC² becomes a state-wide program and human resources for service remains inadequate, MC² was redesigned so that districts can opt to join at two different levels of participation: 1) administrator training to help a district prepare to change its mathematics curriculum, and 2) full district-wide support including teacher and administrator academies and in-school classroom support. The intent is to provide a coordinated statewide cadre of professional development providers in mathematics. This is still in its early stages of development.

**Evaluation Data from MC²**

After triangulating the various sources of data collected for MC², several general conclusions have been reached.

1) MC² services continue to be greatly needed. Budget shortfalls and the mindset that teachers should be in their classrooms teaching, not attending professional development training, has lead to limited opportunities for teacher PD. Field specialists can offer PD during regularly scheduled PLCs or they can model teaching strategies for teachers in their classrooms during school time. In the 2009-2010 school year, MC² provided PD for over 400 teachers from 81 schools in 21 New Mexico districts. Over 2,500 hours of PD were provided. The mean number of hours received by each teacher was 14.9. This does not include the hours spent at the MC² Summer Math Academies.

Data culled from the various data sources collected as part of the MC² evaluation reinforce the teachers’ need for professional development in mathematics.
• Only 17% of the participants in the MC\(^2\) Summer Academies have Bachelor’s Degrees in math or math education; even fewer have Master’s Degrees in those areas (11%).
• Thirty-eight percent of the teachers at the Summer Math Academies have spent five years or less teaching math.
• More than 1/5 of the participants indicated that they were teaching a grade level for the first time in the 2009-2010 school year.
• Forty-nine percent of the Summer Academy participants must prepare for not only math instruction, but also for other subjects they teach.
• An item analysis of the Mathematics Knowledge for Teaching (University of Michigan) teacher responses indicates that many teachers have difficulty with both mathematical content and pedagogical knowledge.

2) The feedback from the recipients of the PD provided by MC\(^2\) is overwhelmingly positive. When participants are asked to rate the usefulness of the PD sessions based on a scale of 1 to 5, with 1 indicating “not useful at all” and 5 indicating “extremely useful,” the mean ratings range from 4.1 to 4.8. Seventy-five percent of the ratings are 4.5 or higher. It should also be noted that more than half of the MC\(^2\) Summer Math Academy participants return each summer to learn more about teaching mathematics.

MC\(^2\) also lead schools to create math leadership teams, including administrators and mathematics leaders, through the Lenses on Learning (LOL) process (Educational Development Cooperation, EDC). Six sessions of LOL training were held in four different regions of the state in 2009-2010. Approximately 120 teachers, principals, and school administrators participated. These sessions culminated in school or district action plans for math. The vast majority of the participants indicated that the LOL sessions were 1) helpful in understanding current research in mathematics education and student mathematic learning and 2) valuable in helping them to gather and analyze school and district data to develop a plan.

3) MC\(^2\) has lead to changes in teacher math content knowledge, classroom practice and student achievement. Teacher gains on the Mathematics Knowledge for Teachers (MKT) instrument were significant after attending the 2010 summer academy, especially gains in Number Concepts and Operations--NCOP (p<0.0001). The focus of the 2010 academies was multiplication, an
integral part of number concepts and operations. Classroom observations, though limited, indicate that middle school teacher classrooms are slowly evolving into more inquiry-based learning environments. Mathematics scores for MC² partner districts on the New Mexico Standards Based Assessment (NMSBA), though mixed, have shown some improvement over the years, especially for economically disadvantaged students.

What We Have Learned from MC²
MC² had evolved from our learning from each year’s successes and challenges. We now know that PD becomes more effective if, in addition to the summer academies, it is delivered routinely during the school year, close to the classroom, and is aligned with what teachers are learning in the summer academies.

Our experience has taught us that teachers must learn mathematical content knowledge in the context of the pedagogy of teaching mathematics. In the early academies, teachers saw the summer academies as a place to learn math for themselves; they didn’t see it as place to learn about how to teach math in the classrooms. Teachers didn’t see or understand the interaction between math and pedagogy. Once the team began to explicitly share and discuss the intentional instructional moves required to teach the math and how teachers might use these strategies in their classrooms, the pedagogy modeled in the summer academies began to transfer to K-12 classroom practice.

Scaling Up Mathematics Achievement (SUMA)
Scaling Up Mathematics Achievement is a research program and a partnership between NMSU and Las Cruces Public Schools (LCPS) to study mathematics teaching and learning in Grades K-8. The five-year NSF-funded grant was awarded to NMSU in fall 2007. This project integrated design-based research into a research partnership between university researchers and mathematicians and a school district leadership team.

As mentioned earlier and as a result of the analysis of the Gadsden Mathematics Initiative, a Building Capacity Model (BCM) was developed. The original BCM which evolved from the GMI is shown as Figure 2 below. It represented the essential elements for developing a district mathematics program for increasing mathematics achievement for all students. The purpose of the SUMA grant was to see if this model could be useful in improving mathematics teaching and learning in a larger, more diverse district.
The grant was written in collaboration with the district but before funding was received the leadership of the district changed completely and a new superintendent and staff became involved. A different kind of partnership evolved and with the help of the SUMA National Advisory Board of experts in mathematics and science education, a focus on design-based research and data-based feedback, and work on developing partnership agreements, the project has resulted in a more complete and dynamic model for building capacity for mathematics achievement. In Figure 3, the BCM is shown in its present iteration.
One of the innovative features of SUMA was the inclusion of two teachers from LCPS to assist the NMSU staff with collecting classroom data. These teacher researchers were also a valuable connection between the school district and the university and helped to bring much needed insight into the organizational and classroom culture of LCPS for the university team. They were able to assist university faculty in bridging theories of mathematics teaching and learning with the reality of classroom implementation.

The teacher researchers also spent much of their time observing math instruction in K-8 classrooms. Almost 3,000 students in 150 classrooms were observed participating in math learning for over 600 hours from spring 2008 to spring 2010. Generally the same classrooms were observed once each semester. To record and report their findings, the teacher researchers used two observation tools: the Classroom Snapshot and a new instrument for classroom observations, Observation of Learning Environments (OLE). Both originally were adapted from the Classroom Observation Protocol (Horizon Research Inc., 2003).

The OLE was developed by the SUMA research faculty (Kinzer, Korn, Wiburg, Virag and Valdez, 2011). The team also received help from national experts in mathematics research and equity issues including Dr. Sara Libienski, Dr. Ed Silver, and Dr. Roberta Shor who served on the SUMA National Advisory Committee. The OLE instrument documents important changes in mathematics learning environments. The theoretical foundation for this instrument includes the work of Tarr et.al. (2008) who found that learning requires both a strong reform curriculum and a standards-based learning environment. One without the other just doesn’t work.

The OLE has five indicators which target the learning environment, the teachers and students role in the math lesson, and the impact of the lesson on students’ learning. In addition to the four key elements listed below, the OLE also looks at the general level of implementation of a mathematics learning environment.

- The teacher’s instructional decisions and questions enhanced students’ conceptual understanding and making sense of math.
- Students were encouraged to ask questions, generate conjectures, offer alternative solution strategies, and show their understanding. Students used multiple perspectives
and strategies, showed math understandings in multiple ways, and asked questions to extend questions.

- Interactions reflected collaborative working relationships among students and with the teacher, showing a classroom climate of trust where the teacher and students could talk with each other, were engaged in explaining and thinking, used math terminology and built knowledge based upon each others’ ideas.
- The teacher displayed an understanding of mathematics concepts. The teachers seemed confident in their own mathematics knowledge, understood the mathematical focus of their lessons, and could differentiate mathematics concepts in the classroom environment.

The OLE has strong Inter-Rater Reliability (Weighted Kappa = .84) and a very strong Internal Structure Reliability (Alpha = .96) (Valdez, 2010). Current work is determining validity. Internal Structure Validity will be a Factor Analysis test and concurrent vailidity with the Horizon Instrument can also be shown.

SUMA Data Chronicling Changes in LCPS
Based on data collected by SUMA, several changes appear to have taken place in math instruction and student achievement in LCPS. First of all, the extensive observations of classroom instruction and learning have indicated positive changes in mathematics learning environments. For each of the four observations done in Grades K-5 between fall 2008 and spring 2010, the percentage of classrooms observed implementing a standards-based learning environment at a proficient or advanced level increased

**Graph 4: LCPS Elementary School OLE Data**

In fall 2008, SUMA Teacher Researchers observed 68 classrooms in Grades K-5.
In spring 2009, SUMA Teacher Researchers observed 136 classrooms in Grades K-5.
In fall 2009, SUMA Teacher Researchers observed 114 classrooms in Grades K-5.
In spring 2010, SUMA Teacher Researchers observed 111 classrooms in Grades K-5.

<table>
<thead>
<tr>
<th>Elementary Schools</th>
<th>Fall 2008</th>
<th>Spring 2009</th>
<th>Fall 2009</th>
<th>Spring 2010</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Level 0) Non Use</td>
<td>7%</td>
<td>8%</td>
<td>4%</td>
<td>5%</td>
</tr>
<tr>
<td>(Level 1) Beginning Step</td>
<td>38%</td>
<td>35%</td>
<td>18%</td>
<td>28%</td>
</tr>
<tr>
<td>(Level 2) Nearing Proficient</td>
<td>34%</td>
<td>27%</td>
<td>39%</td>
<td>23%</td>
</tr>
<tr>
<td>(Level 3) Proficient</td>
<td>19%</td>
<td>26%</td>
<td>32%</td>
<td>30%</td>
</tr>
<tr>
<td>(Level 4) Advanced</td>
<td>1%</td>
<td>4%</td>
<td>8%</td>
<td>14%</td>
</tr>
</tbody>
</table>
As shown in Graph 5 below, the middle school classrooms were observed three times with the OLE and showed a marked improvement during the second observation. The third observation indicated a decline in the use of a standards-based learning environment, but the percentage of proficient or advanced classrooms was still double the initial percentage.

Graph 5: LCPS Middle School OLE Data

In spring 2009, SUMA Teacher Researchers observed 20 classrooms in Grades 6-8.
In fall 2009, SUMA Teacher Researchers observed 17 classrooms in Grades 6-8.
In spring 2010, SUMA Teacher Researchers observed 15 classrooms in Grades 6-8.
Assessment data from the NMSBA indicate that there has been a general improvement in student mathematics achievement from 2008 in the first year of SUMA to the spring of 2010, the end of the third year of SUMA. Table 4 displays the percent of LCPS schools with higher math proficiency in 2010 than in 2008. The mean percent of gain is in parentheses. It is impressive to note that gains were made by all student groups.

Table 4: Percent of LCPS Schools with 2010 Math Performance Higher than Their 2008 Math Performance
(Mean Gain in Percent Proficient or Above in Parentheses)

<table>
<thead>
<tr>
<th>Las Cruces Public Schools</th>
<th>All Students</th>
<th>Caucasian</th>
<th>Hispanic</th>
<th>English Language Learners</th>
<th>Students with Disabilities</th>
<th>Economically Disadvantaged</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elementary Schools (n=24)</td>
<td>79% (+8.8)</td>
<td>77% (+10.2)</td>
<td>79% (+7.1)</td>
<td>76% (+7.4)</td>
<td>77% (+5.7)</td>
<td>87% (+8.4)</td>
</tr>
<tr>
<td>Secondary Schools (n=10)</td>
<td>90% (+8.0)</td>
<td>100% (+9.4)</td>
<td>90% (+7.7)</td>
<td>75% (+6.7)</td>
<td>80% (+2.9)</td>
<td>80% (+9.1)</td>
</tr>
</tbody>
</table>

Source: NMPED School Accountability Reports

Finally, the extensive data feedback provided by the project staff to the district appears to have had an influence on the increased use of data for instruction in the district. A district Co-PI for the grant designed an intensive data-driven decision-making process for all schools in the district based on feeder patterns. District and school administrators, as well as district PD providers, meet every month with university staff to discuss the various sources of student data and the use of appropriate data analysis to make informed decisions.

School principals and their professional development teachers (PDTs)—math/reading coaches—were surveyed three times since SUMA began using the Quality Mathematics Education Matrix (QMEM) (New Mexico Public Education Department, 2006). The QMEM is a rubric that allows school districts to assess their strengths and weaknesses in regards to the quality and implementation of curriculum, professional development, and shared leadership models as they relate to building district capacity for improving mathematics achievement.

In general, the principals and PDTs indicated that in the past three years significant steps have been made in the district’s leadership to get closer to the ideal state of math instruction and learning. The elementary principals and the PDTs perceived large gains in the way district leaders design and monitor a curriculum implementation plan in which all teachers must
participate. In the area of data management structures to monitor implementation of the program and ensure that data collection and analysis is part of the culture of the system, all three sets of respondents indicated that there has been great progress. They also indicated that since fall 2007 there has been a much clearer and more consistent vision of the mathematics program and that vision has been more clearly communicated with all stakeholders.

**SUMA Years Four and Five**
The final two years of data collection and analysis for SUMA will include the use of a mixed effects Hierarchical Linear Model (HLM) which controls for differences in classrooms and schools. Through the HLM the researchers will be able to define which elements of the building capacity model affect achievement. Initial findings just completed indicate that the rating on classroom learning environments with the OLE does seem to contribute significantly to student gains in scores on the annual standardized test. This finding confirms decisions made as the research agenda evolved to make sure that change is occurring in classrooms and not just in teacher professional development. The researchers will also study the interactions between the elements in terms of their effect on the math achievement of student sub-groups.

**What We Are Learning from SUMA**
Working with Las Cruces Public School District, a 34 school urban district with 24,000 students from varied ethnic and socio-economic groups proved to be a very different experience than working in a 20 school rural, homogeneous district of 14,000 students (Gadsden). The very nature and complexity of a larger district with a large central staff oftentimes proved to be challenging. First and foremost, we found that stable leadership that is committed to the new math program is essential for effective change to occur. In Gadsden, there was one specific district administrator who, from the beginning of the grant (and even before) was totally involved and committed to improving the math performance of students in the district. While the SUMA grant was written in collaboration with the Las Cruces district, before funding was received, the leadership of the district changed completely after a new superintendent was hired. We came to realize soon after SUMA began that the policies, structure, and context of the district and community must be given serious consideration when attempting to implement a building capacity model. It became obvious as the grant proceeded that the district-university partnership would have greatly benefitted from
an explicit, agreed upon vision and action plan. Change cannot occur if there is no readiness for change or if a district feels the status quo is fine.

When LCPS first adopted the standards-based mathematics materials for the district, there were some rather strong objections and concerns voiced by some parents. As parents were called together at district and school meetings and given more information about the quality of the mathematics instruction their children would receive, the negative reactions subsided. Early and honest communication with parents is essential whenever major instructional decisions are implemented by a district, especially a district that had never before had a single core math program required in all schools. Ideally this communication should start long before the actual enactment of change.

We have also learned that many teachers in the classroom have not developed sufficient and specialized knowledge to teach mathematics. During classroom observation, mistakes in math content were noted and there were occasions when teachers were unable to respond to student questions because they lacked sufficient knowledge of developmental ideas in mathematics. Intense, systematic professional development is essential for implementing a reform-based mathematics program. This profession development must be for teachers and administrators. Without administrator understanding and support, enactment of a standards-based mathematics curriculum is uneven. There must be a concerted effort to monitor mathematics teaching and learning in the classroom and this cannot be accomplished without teachers and administrators sharing common language and goals.

Research Directions Emerging from MC² and SUMA
This paper argues for taking a system-wide perspective that includes understanding the cultural, political and social pressures in a school district in order to create sustainable change in K-12 mathematics. The researchers have found through extensive observational data from classrooms and yearly analysis of short- and long-term achievement data, that in almost all of our partner districts the patterns of mathematical errors is very similar. Children seem to miss the same kinds of test items regardless of socio-economic or ELL status. While ELL students score at a lower level than mainstream students they are still experiencing difficulty with the same type of item that all students are having trouble with. This reconfirms our commitment to engage in mathematics reform by addressing system-wide approaches and how these district-wide efforts
are needed to increase the achievement of all students. We are continuing to research these related patterns of mathematical understanding and mis-understanding have recently received funding for a new project, *Math Snacks*, that supports the development of multimedia products to address wide-ranging misperceptions of mathematical ideas.

A second weakness that was found in all of the previous programs for mathematical reform is the lack of preparation of teachers to serve as mathematics teachers and leaders. A newly funded program (*MC² Leadership Institute for Teachers*) will provide learning opportunities for teachers who want to become intellectual leaders in their schools. They do not leave their classrooms, but instead attend classes on Saturdays and during the summer and then implement what they have learned in collaboration with district and project staff. Their principals are also required to participate in the institute with the teachers.

**MC² Leadership Institute for Teachers (LIFT)**

*MC² Leadership Institute for Teachers* is an NSF-funded leadership institute for in-service mathematics teachers. As its name indicates, LIFT is an effort to address the lack of strong mathematics teacher leaders in partner districts that was found in all of the previous grant-funded projects. There was a strong need for teachers and principals to understand mathematics content, pedagogy and leadership. To address this need a 2-year institute for professional learning is being designed and implemented by mathematicians, education faculty, and school district leaders. Currently 31 teachers are in the first cohort and a second cohort is planned for years 3 and 4 of the grant.

The LIFT project staff is comprised of three core working groups. The Development Team includes mathematicians and math educators who research and design the course content. The School Support Team partners with teachers in their school to implement the ideas developed in their coursework. These two teams provide the context for the work and basis for our project research. The third team, the Research Team, gathers, analyzes, and shares data regarding the actual change in classroom practice and learning that results from the coursework and school-based support.

The initial cohort of mathematics teacher leaders, from five partner school districts, was selected in 2010. The goals for these mathematics teacher leaders are to: 1) have a deep understanding of
vertical progression and connection of K-12 math ideas; 2) understand and use pedagogical practices that represent a quality mathematics learning environment; and 3) develop the knowledge, skills, and dispositions to become intellectual leaders their districts and school.

The teachers in the cohort, from grades K through high school, study mathematics together. The intention of combining elementary and secondary teachers is for all teachers to understand the development of key mathematical themes. For example, in a recent course on geometry, the LIFT instructors began with an investigation of properties of geometric shapes, both with and without measurement, and ended with a construction of the distance formula based on the Pythagorean Theorem. Teachers were able to see how ideas of length and measurement were instrumental in a child’s understanding of the distance formula. Teachers of later grades learned valuable insights for understanding and correcting student’s difficulties with necessary prior learning.

Learning through vertically aligned ideas represents the current stage of our thinking about mathematics education for teachers. The study of mathematics through vertical trajectories creates a learning environment with multiple entry points, many landing places, and high ceilings. It affords teachers a way of putting their work with students in a bigger context. Our aim is to help our cohort teachers to: 1) develop a deep understanding of the mathematics they teach; 2) understand how the content they teach is connected across grade levels; 3) develop the pedagogical content knowledge to effectively differentiate instruction as different learning needs arise; 4) understand how their instructional decisions are based in different learning theories; and, 5) conscientiously make instructional decisions to match the learning needs of their students.

Evaluation Data from LIFT

LIFT evaluation data has already shown positive changes in teacher mathematics content knowledge and classroom teaching and learning. The first area of improvement was found in teacher gains in mathematics knowledge. Pre and Post mathematics content tests are used each semester to assess teachers’ growth in mathematics understanding on various topics. For example, in fall 2010, the MC² LIFT institute focused on geometry, so a geometry assessment was developed by the LIFT mathematicians in conjunction with the LIFT research team. The same assessment was administered as a pre- and post-test at the beginning of the fall 2010 course and again at the end of the course. The test consisted of eight items which required the LIFT
cohort not only to produce a correct answer, but also to explain their thinking about how they arrived at that answer.

A scoring rubric was developed by the LIFT mathematicians and research team. Each item on the test was worth up to three points—a correct answer was worth one point, an explanation of the answer was worth either one or two points depending on the completeness and quality of the explanation. The maximum number of points that could be earned on the geometry test was twenty-four. Two members of the LIFT research team separately scored the tests using the rubric and sample responses that were provided by the mathematicians. Then the two scorers met to compare their scores. Differences in scoring were minor, and discrepancies were resolved through extensive discussion until a consensus was met.

Twenty-six of the thirty teachers in the cohort (87%) showed gains on the geometry post-test. The total LIFT cohort scored 4.2 points higher on the post-test than on the pretest. The mean score on the pre-test was 14.5 (60% of the total possible score points) compared to a mean of 18.7 (78% of the total possible score points) on the post-test.

All three grade level groups (elementary, middle school, and high school) made gains on the geometry post-test. While the largest gains on the test were made by the elementary teachers (+ 6.4), their initial scores on the pre-test were the lowest of the three groups. Both the middle-school and high-school teachers improved an average of 2.4 points on the post-test. Initially there was over a six point difference between the elementary teachers and the middle- and high-school teachers. That gap narrowed to approximately 2.5 points after the post-test.

For the elementary teachers in the cohort, the pre-test mean was 10.8, while the post-test mean was 17.2. This was a statistically significant improvement, as indicated by a dependent t-test, resulting in $t(12)=5.83$, and $p<.0001$, one-tailed. Similar improvement was displayed by the middle-school teachers, with an increase from 17.5 to 19.9 for pre-test to post-test means. The dependent t-test for this group indicated values of $t(10)=4.50$, and $p=.0006$, one-tailed. Likely due to sample size, the high-school teachers did not show a statistically significant improvement, although their gain was equal to that of the middle school teachers (+2.4), pre-test to post-test means moving from 17.3 to 19.7. In this case, the dependent t-test indicated values of $t(5)=1.14$, and $p=.14$, one-tailed. When combining all 30 subjects that had available data, the pre-test to
post-test mean gains went from 14.53 to 18.67, with $t(29)=6.33$, and $p<.0001$, one-tailed. These results appear to indicate that overall, the participants made substantial improvements on the geometry assessment.

Graph 6 shows the mean percent of score points earned on the geometry pre-test compared to the mean percent of score points earned on the post-test.

The second area of improvement was revealed by classroom observations. The classrooms of all cohort members have been observed three times since the grant began (spring 2010, early fall 2010 and late fall 2010) and will continued to be observed twice each semester for the duration of the program. One of the tools used for these observations is the OLE, which was developed by the SUMA staff.

The initial observations (spring 2010) indicated that 26% of the teachers in the cohort were proficient or advanced in establishing an effective standards based learning environment (SBLE) in their classrooms. This figure rose to 29% in the early fall of 2010. The third observation in late fall 2010 indicated that a much greater percentage, 48% of the teachers in the LIFT cohort, were utilizing characteristics of an SBLE in their classrooms.
What We Are Learning from LIFT

As a result of growth in the mathematicians and educators who have now worked together over the last five years, the Leadership Institute for Teachers was designed to integrate pedagogy and mathematics while focusing specifically on what teachers are expected to teach in their classrooms. LIFT explicitly links what the teacher participants are learning to student math standards and each teacher’s student achievement data. For instance, during our study of geometry, teachers used their own student achievement data to analyze their students’ strengths and weaknesses in geometry. Our development team considered student achievement data in choosing which areas of geometry would be most relevant for study in the institute. As teachers learned about geometric concepts across the K-12 curriculum, they designed an instructional unit to address students’ weakest areas in geometry.

LIFT developers have taken the development of a vertical trajectory in mathematics that evolved in the MC² summer academies as a key component for teacher learning in the LIFT institute. They have also learned to design learning experiences that capitalize on the strengths of both elementary and high school teachers. LIFT development staff has employed intentional strategies to shape a K-12 mathematics professional learning community into a culture of collective trust. Elementary teachers often lack confidence in their mathematics abilities when paired with secondary math teachers. Secondary teachers had a tendency to want to “rescue” the elementary teachers rather that help them develop their own mathematical thinking. The interesting dilemma
is that elementary teachers often were much better able to describe their mathematical thinking from a conceptual framework than secondary teachers who often relied on memorized procedures as part of their solution methods. The development and teaching group employed strategies for classroom discourse in our institute to highlight the richness of elementary teachers’ mathematical thinking. The instructors also needed to intentionally scaffold the class discussions to encourage elementary teachers to share their conceptual understanding and make connections to procedures presented by secondary teachers. The scaffolding of discussion by the mathematicians and mathematics educators proved to benefit both elementary and secondary teachers and has resulted in increased confidence by the elementary teachers. They are now comfortable and confident in working with high school teachers. The high school teachers are now appreciating the elementary teachers search for meaning behind the use of procedures as well as the insights the elementary teachers provide.

LIFT provides a great opportunity to build on continuous feedback from the participants since they meet every other Saturday. Throughout the institute, teacher leaders are asked to provide feedback to the development team about their learning, engagement, and relevance to classroom practice. Strategies for feedback include the following:

- **Feedback on Mathematics Learning/Writing:** Teachers receive feedback on their explanation of mathematical ideas through written comments from project faculty. The process helps teachers strengthen their mathematics and pedagogical content and helps faculty understand the knowledge and learning needs of teachers.

- **Feedback Surveys:** Surveys were administered in the fall to gauge teachers’ perceptions of the relevance of institute learning to their own classroom practice. Teachers were asked to recommend how to strengthen connections between the coursework and classroom practice.

- **Focus Groups:** Three teacher focus groups have been held with the internal and external evaluator to probe for specific strengths and weaknesses of institute design as they relate to teacher learning.

This feedback has been used to reflect on the effectiveness of the institute and redesign institute activities to meet the goals of the project.
LIFT has also learned to provide teacher-to-teacher support systems which seem to be necessary to help teachers redefine themselves as leaders. When the institute first began, teachers were preoccupied with “becoming the math expert” so they could be an effective math leader. However, the project goal is not to develop authority figures but, rather, to develop leadership skills that foster a collaborative professional learning environment. We realized that if we want teachers to develop as teacher leaders who collaborate with others to improve classroom practice, rather than dispensers of knowledge, we needed to model a reflective and collaborative learning environment for teachers in the cohort.

Designing teacher-to-teacher support systems has helped open up conversations about classroom practice and foster a system where teachers are accountable to one another, rather than to project faculty. Two models we are using to support teacher-to-teacher accountability are the “Teacher Learning Community” (TLC) model from Dylan Wiliam (2007) and Lesson Study (Lewis, 2002). In the TLC model, teachers meet as a learning community in small groups every two weeks. In these groups, teachers share learning from practice. They set goals for implementing a change in practice and gather evidence of how their new instructional strategy or activity impacted student learning. In the Lesson Study model, teachers collaborate in action research to design a classroom lesson, gather data through observation in one another’s classrooms, and reflect on how the mathematics content and instructional design impact student learning of the material. Both models have served to make classroom practice and teacher learning a collaborative and public process.

Since our ultimate goal is to improve classroom practice, we needed tools to highlight the strengths and weaknesses of classroom practice among our cohort. It is difficult (if not impossible) for teachers to reflect on their own practice without tools that provide an objective viewpoint. Therefore, as part of school support, tools were developed and adapted for teachers to engage in self reflection. One tool we used is the Cognitive Demand Classroom Self-Reflection (2009) adapted from the Lincoln Achievement in Mathematics Partnership Project, U.S. Department of Education, Math-Science Partnership Project. Teachers video-taped their classroom practice and used the tool to reflect on the strengths and weaknesses of their practice in relation to students’ engagement doing mathematics. The LIFT school support team used the Shared Classroom Experience Protocol (Horowitz, Bradley, and Hoy, 2011). This tool is used
by teachers and project faculty to study together student learning and reflect on strategies to engage students in the mathematics.

The development of three teams (development, support, and research) has been useful in LIFT; however, we have also found it essential that the three teams meet frequently with each other in order to clarify our goals for the teachers and provide a better support structure for their learning. Comparison of classroom observations of cohort members from spring 2010 to early fall 2010 showed little change. However, with greater collaboration between teams, we saw positive change in OLE classroom ratings in late fall 2010. The research team has also shared data regarding project impact with teachers so the teachers can see how they are progressing as a cohort. Sharing data helps teachers to understand the project goals and how their own personal learning and classroom instruction impacts the cohort data. The practice of sharing research data is promising in moving cohort teachers from focusing on individual success to seeing their individual efforts as supporting success of the project as a whole.

**Math Snacks**

*Math Snacks* is a five-year research and development grant which was awarded to NMSU by NSF in fall 2009. The grant is a little different from many of the earlier programs reviewed in this paper. It is a development grant and the first two years are being spent on the development of animations, games and tools designed to address the conceptual learning gaps that had been uncovered in the earlier analysis of 24,000 student scores on the New Mexico Standards-Based Mathematics Assessment (NMSBA). The NMSBA is a high-level test which is closely aligned with the National Assessment of Educational Progress. It is also quite different from the tests in many states since 50% of the score points on the test come from student responses to short answer and open-ended questions. Thus it is possible in looking at the results to see not only right or wrong answers but also problems students had in answering more open-ended questions.

The purpose of *Math Snacks* is to address gaps in conceptual mathematics understanding with innovative media. The grant targets students in Grades 6 and 7. Data from the mathematics programs mentioned earlier in this paper were used to determine the gaps to be addressed: student achievement data (NMSBA) and teacher content knowledge data (MKT) data came from MC² and SUMA; data about teacher classroom practice came from MC², SUMA, and LIFT. We had enough
data about common areas of difficulty that we were confident in planning the goals for the Math Snacks products to be developed.

However, there was still a choice of which conceptual gaps were the most important. There were numerous different gaps we could have addressed just in terms of students having difficulty with test items. Therefore the Math Snacks team developed some additional criteria for determining which gaps would be addressed in Math Snacks. We asked ourselves three questions:

1. Is the concept to be learned one that is built upon in future mathematics learning? In other words, if students never understand a concept, e.g. the distributive property, the idea of units, will they have trouble in future math classes?
2. Is the concept one that we can develop in a way that allows multiple points of entry including an easy entry point and a high ceiling. Is the concept extensible?
3. Is the concept one that technology can help us teach? Will students be helped by the affordances of technology such as graphical interfaces, use of sound and color, and the ability to manipulate objects, to better understand a mathematical idea?

The mathematicians and educators on the team struggled for the first year to decide on the most important concepts to be addressed. We also had to learn to work with the multimedia development team so that ideas that we thought up were possible in the minds of the programmers and artists. Now in the second year, we have developed an interactive process for development in which members of all the teams work together in small groups to move from mathematical goals (in response to selected gaps) to solid proposals for Math Snacks products.

This interactive work is very powerful and expands earlier work between mathematicians and educators to a third group, educational software developers. Now, in year two we have one concrete set of math goals for all three years of development. All of the goals are grounded in several further principles for development. The product must not look like math. It should not look anything like a traditional mathematics workbooks. When possible we include very few numbers in our videos and games, using graphics and inter-action to reinforce the math concept. In addition, as much as possible we try to place mathematics in the context of engaging real world settings. This has led to interesting products such as the short video, The Bad Date, in which ratios are
explored in the context of how many words each person uses in a conversation. Examples of *Math Snacks* can be found at mathsnacks.org.

Another principle behind *Math Snacks* is that our products are designed to be conceptual. The vast majority of math software is designed for practice. There is nothing wrong with this use of technology, but there is also a need to provide students with a different kind of introduction to key mathematics ideas. Our intent is to engage students in conceptual thinking. This is not easy and requires a lot of time and sometimes many false starts and abandoning of initial ideas. First, all of the products are tested by students in the Learning Games Lab during the summer and also after school. The testing of early prototypes avoids making anything that students just don’t like. Second, the *Math Snacks* products are tested in classrooms with collaborating teachers. In year two of the project we are getting this classroom testing underway and have fifteen collaborating teachers of 5th-8th grade students.

In addition, we decided last summer to provide a *Math Snacks* Summer Camp as a means for both professional development for collaborating teachers (an opportunity to learn what the kids are learning from using *Math Snacks*), and further pilot testing of the products. The *Snacks* summer camp was a spectacular success as demonstrated by video interviews and student and teacher products. All of the students wanted to come back again this summer for another math camp. Having professional development for teachers in the context of teaching to students in the morning and debriefing in the afternoon worked extremely well. This approach is part of our move toward affecting what happens in classrooms. The collaborating teachers were asked to plan the lessons for the students and teach the lessons in teams to the students in the morning. Then they met with staff and discussed what the students were learning and how the lessons went and how they could be improved for the next day. Each team of teachers worked with one concept, for example, ratios and one or two *Math Snacks*, in this case a *Dodge Ball* animation and *The Bad Date*. They planned additional learning activities for students around that math concept. This may have grown out of former experiences with doing Lesson Study for an early project funded in the 1990s. In that project we did the common summer academy for teachers in mathematics and then visited them in their classrooms during the year and found no evidence of anything they had learned being provided to their students.

The developed products for *Math Snacks* by the end of pilot testing in year three will include:
• animations and games designed for use by middle-school learners in class, during extended
learning periods, or at home;
• companion print materials as needed to assist learners in applying conceptual understanding to
learning; and
• short video clips documenting best practices by exemplary teachers using the developed
materials with students, setting up the learning materials, and reviewing products with class
discussion.

Pilot testing of Math Snacks modules will occur in classrooms this year and next. When possible
we will also test the use of Math Snacks in after-school and informal learning programs since the
intention is to provide a tool that students can use any time and in any place where they have
access to the Internet. In years four and five we will conduct a set of randomized control group
trials to see if using these products together with web-based professional development support
(teacher guides, student guides, ELL materials, videos of teachers and kids using Math Snacks) can
make a significant difference in student learning of the targeted key mathematics concepts.

Conclusions
Looking back over our research projects and the developing research agenda, several themes
emerge. These include the need for
• changing forms of professional development for teachers;
• integrating reform with the district management and instructional systems;
• developing relationships across disciplines and across university-district partnerships; and
• evolving mathematics frameworks for learning mathematics knowledge for teaching.

Changing Forms of Professional Development
One of the common themes that developed in this research agenda is a recommendation to
increase the connection between teacher professional development and student learning.
Traditional summer academies in which teachers come and learn but seldom bring the learning
back in to the classroom are no longer desirable as stand-alone projects. However, summer
academies can be useful when connected to follow-up support at the school site either by district
math coaches or grant staff. The current work in Math Snacks involving professional
development with both teachers and students may be a model worth considering especially since
there are limited school budgets, and administrators and teachers are reluctant to take any time away from teaching. Providing professional development for teachers by having them engage in teaching seems like an especially powerful form of professional development and in many ways the teachers learn as much or more mathematics than the students. There is also continuing professional development work in MC\(^2\) and in the MC\(^2\) LIFT projects that involves going into classrooms and modeling lessons for teachers, team teaching with teachers, and then coaching teachers on their lessons.

**District-Wide Approaches**

In order to increase the transfer of learning from teacher professional development to student learning opportunities it is also necessary to involve the whole district and the district administrators so there is support for effective mathematics teaching and learning. While some people have succeeded in working with one school, it is often the case that after a certain principal or key player leaves that school, the reform is not maintained by the district. Of course the best example we have of sustainability is the Gadsden district in which the student scores in mathematics continue to rise even several years after funding has ended. Building capacity for underrepresented student groups to excel in mathematics has been a national challenge, as has the need to increase mathematics achievement for all students. While many workshops offer isolated approaches such as language learning strategies, or technology-based *silver bullets* without regard to what happens away from the workshops, the research agenda that has been articulated in this paper addresses the achievement gaps and problems in mathematics education through a systems-approach that includes socio-cultural perspectives and critical partnerships.

**Building New Kinds of Relationships**

It is clear that one of the strengths of our research in mathematics education reflects the development of unusually strong relationships between groups that have quite different cultures and ways of knowing and working. The most obvious example is our success in building partnerships between public schools and universities. The authors of this paper have had a history of doing research in the applied area of classrooms and learning. In addition, three of the authors spent 10 to 25 years working in schools as administrators, evaluators, and mathematics teachers before working in academia. More unusual is the strong working relationship between educators, educational researchers and research mathematicians. While we are not sure why this
has worked so well at our university, one reason, of course, was that both the educators and the mathematicians have a common interest in learning and studying learning. As our work has evolved, the mathematicians might be the first people to ask, “How can we teach this in a way that allows it to be easily accessible to the learner?” or “How can we use graphics or animation to help students understand the underlying math content?” And the educators might be the first to try to understand the pre-requisite tasks in mathematics in order for students to master algebraic equations.

In addition to working across disciplines, we may be further influenced by the university in which we work. New Mexico State University is a Hispanic-serving institution and is dedicated to serving the students, many of whom are first-generation college students. The university itself takes pride in supporting these first-time college students and allowing them to gain access to college and success in college leading to careers and success in life.

Mathematics Knowledge for Teaching

Finally, we have in common an ability to conceptualize problems in terms of systems and systems thinking. It is perhaps this ability that has led us to think of the developing system of mathematical ideas as key to helping teachers learn how to teach. Looking at systems solutions to problems like students not being well-prepared in mathematics may have helped us avoid the phenomenon of going into a school, doing some research, gathering some data and then leaving an unchanged school. The authors of this paper were stimulated by the AERA call for papers to put together a long-term view of how our mathematics education research group has developed and increased our capacity to provide the complex systems approach we believe is required.
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