Math 331 Project 1

This project is due on 10 October 2007 in class. You are to work in groups of 2 or 3 people. You are to turn in one typed paper per group. Any mathematical expression or picture can be hand written.

The purpose of these problems is to obtain a relationship between the length of a code, the number of codewords, and the distance of the code. This relationship is given in Problem 4. The purpose of the first three problems is to give you the information necessary in order to solve Problem 4.

Let $v \in \mathbb{Z}_2^n$ be an arbitrary word. If $r$ is a nonnegative integer, define

$$B_r(v) = \{w \in \mathbb{Z}_2^n : D(w, v) \leq r\}.$$  

This is called the ball of radius $r$ centered at $v$. Let $C$ be a code of length $n$.

1. If $r$ is a nonnegative integer and $v$ is a word in $\mathbb{Z}_2^n$, prove that $v \in B_r(v)$.

2. Let $r \leq n$ be a positive integer and $v$ a word of length $n$. Prove that the number of elements in $B_r(v)$ is

$$\binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{r}.$$  

You may use without proof that, for each integer $i$ with $0 \leq i \leq n$, the number of words a distance exactly $i$ from $v$ is equal to the binomial coefficient $\binom{n}{i}$.

3. If $C$ is a $t$-error correcting code, prove that $B_t(v) \cap B_t(w) = \emptyset$ for all pairs of distinct codewords $v$ and $w$.

4. If $C$ is a $t$-error correcting code and $M$ is the number of codewords in $C$, prove that

$$M \cdot \left(\binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{t}\right) \leq 2^n$$