Homework #3, due Friday 16 February

- Page 16, Problem 1

- Let $G$ be the symmetry group of the whale picture. Describe the action of $G/T \cong \langle r \rangle$ on $T$, via the action given in Example 2.20. That is, write down a simple formula for how $r$ acts on each translation $\tau_v \in T$.

In the following problems, $G = N \times_{\varphi} H$ for some groups $N$ and $H$, and $\varphi : H \to \text{Aut}(N)$ is a group homomorphism. Let $N' = \{(n, 1) : n \in N\}$ and $H' = \{(1, h) : h \in H\}$. The \textit{trivial homomorphism} between groups $A$ and $B$ is the function $f : A \to B$ for which $f(a) = 1$ for all $a \in A$.

- If $G = N \times_{\varphi} H$, prove that $H'$ is normal in $G$ if and only if $\varphi$ is the trivial homomorphism.

- If $\varphi$ is nontrivial, prove that $G$ is not Abelian.

- Prove that $N'$ and $H'$ are subgroups of $G$ with $N' \cong N$ and $H' \cong H$. Furthermore, prove that $G/N' \cong H$.

- Prove that $(h, 1)(n, 1)(h, 1)^{-1} = (\varphi(h)(n), 1)$ for each $h \in H$ and $n \in N$.

(Compare the statement of this problem to that of Problem 1 on Page 16.)

- \textbf{Extra Credit:} Page 16, Problem 3. Moreover, show that if $G$ is a semidirect product of $G_0$ and $N$, then the $x_\sigma$ can be chosen so that $f(\sigma, \tau) = 1$ for all $\sigma, \tau \in G_0$. 