Homework #4, due Friday 2 March

1. Let $r, s$ be rotation about the origin.
   
   (a) Prove that $r \circ s = s \circ r$.
   
   (b) Let $(r, v)$ and $(s, w)$ be rotations (so $r, s$ are both rotations about the origin). Determine a condition on $v, w$ in terms of $r, s$ so that the two rotations have the same center. (Recall Lemma 2.1.)
   
   (c) Let $g = (r, v)$ and $h = (s, w)$ be rotations about different centers. Prove that $ghg^{-1}h^{-1}$ is a nontrivial translation.

2. Let $f$ be reflection about the line through the origin which makes an angle of $\theta$ with the $x$-axis. Show that $f$ is represented by the matrix
   
   \[
   \begin{pmatrix}
   \cos 2\theta & \sin 2\theta \\
   \sin 2\theta & -\cos 2\theta
   \end{pmatrix}.
   \]

3. Let $f, g$ be reflections about lines through the origin. Prove that $f \circ g$ is a rotation about the origin. Moreover, if $\theta$ is the angle between the reflection lines, prove that $f \circ g$ is a rotation by $\theta/2$ (either clockwise or counterclockwise).

4. Let $r \neq I$ be a rotation about the origin, and let $f$ be a reflection about a line not passing through the origin. Prove that $r \circ f$ is a nontrivial glide.

5. Write the symmetry group of the snake picture as a union of cosets of the translation subgroup $T$. Make sure to explicitly identify the coset representatives. Give some justification for why you know your description is correct.