

GENERALIZED IMPLICATIVE STRUCTURES

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Abstract. In this work we shall introduce and study the class of bounded distributive lattices endowed with a binary mapping between the bounded distributive lattice and the set of the ideals. This class of structures is a generalization of the bounded distributive lattices with a weak implication, introduced by S. Celani and R. Jansana [1]. A generalized implication on a bounded distributive lattice \mathbf{A} , is a mapping

$$\Rightarrow: A \times A \rightarrow Id(A)$$

where $Id(A)$ denotes the set of all ideals of \mathbf{A} , such that verifies the following conditions:

C1 $(a \Rightarrow b) \cap (a \Rightarrow c) = a \Rightarrow (b \wedge c)$.

C2 $(a \Rightarrow c) \cap (b \Rightarrow c) = (a \vee b) \Rightarrow c$.

C3 $(a \Rightarrow b) \cap (b \Rightarrow c) \subseteq a \Rightarrow c$.

C4 $a \Rightarrow a = A$.

A generalized implicative structure is a pair $\langle \mathbf{A}, \Rightarrow \rangle$, where \mathbf{A} is a bounded distributive lattice and \Rightarrow is a generalized implication defined on A . We shall prove that exists a topological duality between this class of structures and Priestley spaces with a binary relation satisfying certain conditions. We will give appropriate concepts of homomorphism and congruence in these new structures.

References

- [1] S. Celani and R. Jansana. *Bounded distributive lattices with strict implication*. Preprint.