January 24

Loans
If you borrow money, how is the monthly payment determined?

What does it even mean to say you get a car loan at 6% APR?
APR = annual percentage rate

Say you get a $20,000 loan at 6% per year for 5 years.
Two scenarios:

1. The company invests the $20,000 at 6% per year, compounded monthly. It would have $26,977 after 5 years.

2. The company gives you the loan, and then invests each payment you make at 6% per year, compounded monthly.
To say you have a 6% loan for 5 years means the loan company would have the same amount of money in each of the two scenarios.
Loan

first pmt

last pmt
value at end of loan:
last payment: $P$
second to last: $P(1+r)$
third to last: $P(1+r)^2$

etc.

$L(1+r)^n$ is the end value if $L$ invested for $n$ periods.
So, the payment satisfies the equation

\[ L(l+r)^n = P + P(l+r) + \ldots + P(l+r)^{n-1}. \]

Fortunately, mathematicians have studied these sorts of expressions, and have found formulas to simplify the right hand side. One can then get the following formula:
Loan Formula

\[ P = \frac{L \times r}{1 - (1+r)^{-n}} \]

- P is the monthly payment
- L is the loan amount
- r is the interest rate per month
- n is the number of months
On a calculator, enter

\[ L \times \frac{r}{(12 \times (1 - (1 + (r/12))^{-m}))} = \]

where \( r \) is the yearly interest rate, converted to a decimal (e.g., 5% is entered as .05) and \( m \) is the number of months.
Questions

• What is the monthly payment for a $20,000 car loan at 6% APR for 5 years?

• What if the loan is 8%?

• An S-600 Mercedes lists at $150,000. What would your payment be if you got a loan for the full amount at 6% for 5 years?
Answers

• $386.66

• $405.53

• $2,899.92
How much would you pay in total in all three cases?

1. $23,199

2. $24,332

3. $173,995
Home Loans

With home loans typically for 30 years, interest is much more an issue than for car loans.

Getting a 15 year loan can save a huge amount of money!
Questions

If you borrow $150,000 at 4% per year for 30 years, what is your monthly payment?

What would your monthly payment be at 3%?

At 5%? What about 10%?
3% per year: $632.41

total payments: \[632.41 \times 360 = 227,666\]

4% per year: $716.12, total payments: $257,804

5% per year: $805.23, total payments: $289,884

10% per year: $1,316.36, total payments: $473,889
At 5% per year, you end up paying nearly as much in interest as the value of the loan.

Going from 5% to 3% annual interest would save over $60,000 during the life of the loan, and about $170 per month.

Going from 5% to 10% would cost about $185,000 extra for interest!
Question

How much would your monthly payment be if you borrowed $150,000 at 5% per year, but got a 15 year loan?
monthly payment: $1,186.19

total payments: $213,514

You would save about $75,000 over the life of the loan with a 15 year loan rather than a 30 year loan.
Amortization Tables

One aspect of loans is that early in the loan period, much of the monthly payment goes toward interest, rather than principal. A loan schedule table, or amortization table, will list how much of each payment goes toward interest and to the principal.
You can find loan schedule tables in the spreadsheet InterestCalculator.xls available on the course website.

For example, on a 30 year house loan, nearly all of the initial payments go toward interest.
Credit Cards
Credit cards work the same as loans. The main difference is the high interest rate most charge. Interest rates up to 20% per year are common.
Suppose you have a card which charges 16% per year and a $10,000 credit limit. If you charge $10,000 worth of stuff and pay the $150 minimum payment each month, what happens?
At 16% interest per year, on $10,000 you owe 10000 * .16/12 = $133.33 in interest the first month. If you pay $150, you will have a balance of $10,000 + 133.33 - 150 = $9,983.33 the next month.

Doing the same sort of calculation over and over again will give the following table:
<table>
<thead>
<tr>
<th>Month</th>
<th>Balance</th>
<th>Interest</th>
<th>Payment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$10,000</td>
<td>$133.33</td>
<td>$150</td>
</tr>
<tr>
<td>2</td>
<td>$9,983.33</td>
<td>$133.11</td>
<td>$150</td>
</tr>
<tr>
<td>3</td>
<td>$9,966.44</td>
<td>$132.89</td>
<td>$150</td>
</tr>
<tr>
<td>4</td>
<td>$9,949.33</td>
<td>$132.66</td>
<td>$150</td>
</tr>
<tr>
<td>5</td>
<td>$9,931.99</td>
<td>$132.43</td>
<td>$150</td>
</tr>
<tr>
<td>6</td>
<td>$9,914.41</td>
<td>$132.19</td>
<td>$150</td>
</tr>
<tr>
<td>10</td>
<td>$9,841.75</td>
<td>$131.22</td>
<td>$150</td>
</tr>
<tr>
<td>15</td>
<td>$9,745.33</td>
<td>$129.94</td>
<td>$150</td>
</tr>
<tr>
<td>20</td>
<td>$9,642.30</td>
<td>$128.56</td>
<td>$150</td>
</tr>
</tbody>
</table>
After 20 months, you will have paid $3000, but only about $360 of the principal, and you could not be buying much more on the card.

This calculation is in InterestCalculator.xls on the course website.