January 29

Annuities
An annuity is a repeating payment, typically of a fixed amount, over a period of time. An annuity is like a loan in reverse; rather than paying a loan company, a bank or investment company pays you the monthly payment.
There are two typical calculations for annuities.

1. Paying into an annuity
2. Collecting from an annuity
Paying into an annuity means investing or saving money in order to receive an annuity in the future.

Collecting from annuity then happens once you have invested enough money to receive an annuity.
Paying into an Annuity

Suppose you need to have $100,000 saved 20 years from now. If you can invest at 6% per year, how much do you need to put away each month?

Each monthly investment will be collecting interest; the earlier payments will collect more.
If $P$ is your monthly investment, $r$ the monthly return rate, and $m$ the number of months you invest, then the amount of money you will have after $m$ months is

$$F = \frac{P \times ((1 + r)^m - 1)}{r}$$
If you know how much you will need to have, you can solve this formula for how much you need to invest each month:

\[ P = \frac{F \cdot r}{((1 + r)^m - 1)} \]
To come back to the original question, if you need to have $100,000 in 20 years and you can make 6%, then the amount you need to invest each month is

$$100000 \times \frac{.06/12}{(1 + .06/12)^{240} - 1}$$

which is $216.43.
Collecting from an Annuity

If you are paid an annuity at a monthly interest of \( r \) (converted to a decimal) for \( m \) years, and you’ve invested \( L \) dollars, your monthly income \( P \) will be

\[
P = \frac{L \cdot r}{1 - (1+r)^{-m}}
\]
This is the same formula as for loans, because an annuity is really a loan in which you are the lender.
If you have invested $200,000 and receive an annual rate of return of 7%, how much will you receive each month if you get payments for 20 years?

What if you want payments for 30 years? What about for 40 years?
Answer

Using the loan formula, the values are:

$1550.60 a month for 20 years

$1330.60 a month for 30 years

$1242.86 a month for 40 years
Perpetual Annuity

Annuities in which there is no ending date for the payments are called perpetual annuities. You can think of these as going on forever (although they really don’t).
The formula for the monthly payment from a perpetual annuity is very simple, if you have $L$ dollars invested and you receive a monthly interest rate of $r$, then

$$P = L \times r$$

determines your monthly payment.
Entering in a Spreadsheet

You can enter a formula with numbers more or less as they would be written in these slides. However, you can also use variables. We’ll look at examples of each.
To calculate $100 \times (1 + .05/12)^{240}$, you can type

$$= 100 \times (1 + .05 / 12)^{240}$$

in a cell.

You will get 271.26 (rounding to two places)
You can use cell references for variables. Each entry is determined by a column and a row. An example of a cell reference in Excel is B4; this refers to column B and row 4.
If you want to enter the formula
\[ P \times (1 + r / 12)^n \]
where \( P \) is in B5, \( r \) is in C5, and \( n \) is in D5, you enter, in some cell,

\[ = B5 \times (1 + C5 / 12)^{D5} \]

and hit return.
The advantage of doing this is that you can change the contents of any of the cells and the value of the formula will be automatically recomputed without having to reenter the formula.

This is how I entered information into the file InterestCalculator.xls.
Summary of Interest Rates

- Compound Interest
- Inflation
- Loans
- Paying into an Annuity
- Collecting from an Annuity
Compound Interest

\[ F = P \times (1 + r)^n \]

where,

\( F \) = future amount

\( P \) = principal (initial investment),

\( r \) = interest rate (converted to decimal)

\( n \) = number of years
If $P$ is the present value of money, at an inflation rate of $r\%$ per year (made into a decimal), the equivalent value $n$ years later is

$$F = P \times (1 + r)^n$$
Loan Formula

\[ P = \frac{L \times r}{1 - (1 + r)^{-n}} \]

- **P** is the monthly payment
- **L** is the loan amount
- **r** is the interest rate per month
- **n** is the number of months
Paying into an Annuity

If $P$ is your monthly investment, $r$ the monthly return rate, and $m$ the number of months you invest, then the amount of money you will have after $m$ months is

$$F = \frac{P \times ((1 + r)^m - 1)}{r}$$
Collecting from an Annuity

If you are paid an annuity at r% monthly interest for m years, and you’ve invested F dollars, your monthly income $P$ will be

$$P = \frac{L \cdot r}{1 - (1 + r)^{-m}}$$