15 February 2012

CRACK

BOOM

WHOA! WE SHOULD GET INSIDE!

IT'S OKAY! LIGHTNING ONLY KILLS ABOUT 45 AMERICANS A YEAR, SO THE CHANCES OF DYING ARE ONLY ONE IN 7000,000. LET'S GO ON!

THE ANNUAL DEATH RATE AMONG PEOPLE WHO KNOW THAT STATISTIC IS ONE IN SIX.
Last time we conducted several probability experiments. We’ll do one more before starting to look at how to compute theoretical probabilities.
If you flip two coins, do you think it is equally likely to get two heads as it is to get a heads and a tails?
A Final Coin Flip Experiment

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Flip a pair of coins 20 times and record how many times you got 2 heads and how many times you got one of each. You don’t need to keep track of how many times you got 2 tails.
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Enter the number of times you got 2 heads with your clicker.
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Now enter the number of times you got one of each.
Are you surprised by the class’s results? It turns out that the probability of getting one of each is twice as much as getting two heads. The probability of getting one of each is 50% while the probability of getting two heads is 25%.
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Because we can view this experiment as having three outcomes, 2 heads, 2 tails, one of each, many people would expect each to have a 1/3 chance of happening.

We will now simulate this activity with a spreadsheet, Two Flips.xlsx.
Calculating Probabilities

We’ve looked at several probability experiments and have computed some experimental probabilities. How does one go about calculating theoretical probabilities? For example, what calculation did de Méré do to see that the rolling a pair of dice 24 times was slightly less than 50/50 to get a double six?

We’ll start to look at some ideas for computing probability. The first idea we will look at is the basis of most probability computations, and virtually all we will consider.
A very common situation in probability is to have all possible outcomes be equally likely. Examples are flipping a (single) coin and rolling a (single) die. The two outcomes of the flip, heads and tails, each have a 50% chance of occurring. Each of the 6 outcomes of rolling a die have a 1/6 chance of happening.
Equally Likely Events

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When we have equally likely events, computing probability is simpler. To compute the probability that something happens, you can then use the formula

\[
\text{probability} = \frac{\text{number of ways the outcome can occur}}{\text{total number of outcomes}}.
\]
An Example using a Spinner

Respond to each question by entering a decimal with your clicker.

1. If you spin this spinner, what is the probability you land on blue?
   
   \[ \frac{1}{5} = 0.2 \]

2. What is the probability you land on yellow?
   
   \[ \frac{2}{5} = 0.4 \]

3. What is the probability you don’t land on red?
   
   \[ \frac{4}{5} = 0.8 \]
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If we interpret an outcome as landing in one of the five wedges, then there are 5 equally likely outcomes. With this interpretation, to answer the third question, we have to recognize that 4 of the outcomes represent not landing on red. That is, not landing on red is the same as landing on either blue, yellow, or green.
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A perfectly good interpretation would be to list outcomes as the different colors. If we do this, then the outcomes are not equally likely. It is usually much simpler to interpret outcomes in a way to make them equally likely (if possible).
If you can model a situation with equally likely outcomes, then writing out all outcomes is the most basic way to compute probabilities, but is often effective.
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For example, if you flip a single coin, you have two equally likely outcomes:

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If you roll a single die, you have six equally likely outcomes:

1, 2, 3, 4, 5, 6
Many situations in probability can be viewed as multi-stage situations. For example, when flipping two coins, you can think about it as flipping one, then flipping the other (even if you flip simultaneously). Similarly rolling two dice can be viewed as a two-stage situation. The problem we started to discuss on Monday, rolling a pair of dice 24 times, can be viewed as a 24-stage situation (or a 48-stage!).
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For example, if we flip two coins, we can list the outcomes as

\[ HH, \quad HT, \quad TH, \quad TT \]

Where \( H = \text{heads} \) and \( T = \text{tails} \) and by thinking of \( HT \) as flipping heads, then tails, while \( TH \) represents flipping tails, then heads.
We could also model this situation by writing the outcomes as

\[
\text{two heads, two tails, one of each}
\]

While this is fine, the outcomes are not equally likely, as our simulations have shown. It is for this reason that the first way of modeling flipping two coins is more convenient.
The situation of rolling two dice is similar, but there are more outcomes. If we think of this as a two-stage situation, rolling the first die, then the second, we can list the outcomes in several ways. One way is with the following table.
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<table>
<thead>
<tr>
<th>die 1</th>
<th>die 2</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>(1, 1)</td>
<td>(1, 2)</td>
<td>(1, 3)</td>
<td>(1, 4)</td>
<td>(1, 5)</td>
<td>(1, 6)</td>
</tr>
<tr>
<td>2</td>
<td>(2, 1)</td>
<td>(2, 2)</td>
<td>(2, 3)</td>
<td>(2, 4)</td>
<td>(2, 5)</td>
<td>(2, 6)</td>
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</tr>
<tr>
<td>3</td>
<td>(3, 1)</td>
<td>(3, 2)</td>
<td>(3, 3)</td>
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<td>4</td>
<td>(4, 1)</td>
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<tr>
<td>5</td>
<td>(5, 1)</td>
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<td>(5, 4)</td>
<td>(5, 5)</td>
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</tr>
<tr>
<td>6</td>
<td>(6, 1)</td>
<td>(6, 2)</td>
<td>(6, 3)</td>
<td>(6, 4)</td>
<td>(6, 5)</td>
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<td></td>
</tr>
</tbody>
</table>
Here is another way to list the 36 outcomes of rolling two dice, and keeping track that we are considering one die as the first die and the other as the second.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>(1,1)</td>
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<td>(1,5)</td>
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<tr>
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<td>(1,4)</td>
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Probability Trees

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By realizing that on each flip, \( H \) and \( T \) are equally likely, we can see that each branch of this tree represents a \( \frac{1}{4} \) chance of occurring. The four outcomes are equally likely.

From this we see that there is a 50% chance of getting one of each when flipping twice but only a 25% chance of getting two heads. This answers why getting one of each is twice as likely as getting two heads.
A probability tree can also have probabilities listed on it. Typically one writes the probability of an outcome for each stage.
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For example,
We’ll discuss how listing probabilities on the branches can be used to compute the probability of multi-stage situations on Friday. This will be helpful when outcomes aren’t equally likely.
We can draw a probability tree for rolling two dice and thinking about it as a two-stage situation. We have six branches for the first roll since there are six outcomes. There are six outcomes for the second roll for each possibility of the first roll.
First Roll

Second Roll

(1, 1)
(1, 2)
(1, 3)
(1, 4)
(1, 5)
(1, 6)
(2, 1)
(2, 2)
(2, 3)
(2, 4)
(2, 5)
(2, 6)
(3, 1)
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On Friday we will use probability trees to answer more sophisticated questions, including de Méré’s question. We’ll see that we can use probability trees when we don’t have equally likely outcomes.
If you spin this spinner, what is the probability that you land on either blue or green?

A 1/5  
B 2/5  
C 3/5  
D 4/5  
E 5/5