Expected Value

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We’ll start today with a video of playing a poker hand.
Pot Odds

Suppose you are playing heads up with one other player, and it is down to the last bet. Suppose there is $100 in the pot and your opponent has made a $10 bet. You are wondering if you should call the bet or fold. You estimate you have a 5:1 chance of winning the hand. What do you do?
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What you do is calculate pot odds. This is the ratio of pot size to bet. In this example it is 100:10, or 10:1. If your odds of winning are better than 10:1, which is the case here, it is to your advantage to call the bet.
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A Data blew it! His chance of winning was way better than 1 out of 13 times.
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Another way it is used is to try to get people to make bad bets. By betting enough so that your opponent’s pot odds won’t be good enough to warrant betting, you will either get them to fold or to make a bad bet. This is a sophisticated play, but very good players use this regularly.
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Another aspect of this idea is how payouts are determined by casinos. Roughly, if you bet on the outcome of a game, such as the Super Bowl, the payout is related to the estimated probability that a given team will win. The more likely a team is thought to win, the lower the payout.
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In order to use this idea, you must be able to compute if you’re more likely to make money than lose money with a given action. This leads us to the idea of expected value.
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For example, suppose you bet $1 on whether the flip of a coin comes up heads. You win $1 if the coin comes up heads. You lose your bet if it comes up heads. If you play this game many many times, the most likely outcome is to break even; you expect to win 50% of the time and lose 50% of the time. Since you win or lose $1 each time you play, neither person playing has an advantage.
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Your expected value, or expected amount of winnings, is then 0 for each bet.
Let’s look at another example. Suppose we spin the following spinner.

Suppose you play a game where you win if the spinner lands on blue and you lose otherwise. For a $1 bet, if you win you receive $3. Is this a good game for you to play?
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The probability of winning is $1/5$. That means, on average, for every 5 times you play you win once. So, on average, every 5 times you play, you win 1 time and lose 4 times. If you bet $1 each time you’ll win $3 and lose $4, on average. So, you will lose $1 out of every 5 bets, on average, or $0.20 per bet.
In other words, the expected value of playing is \(-$0.20\) per $1 bet. It is negative because it is more likely for you to lose than to win.
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In other words, the expected value of playing is $-0.20$ per $1$ bet. It is negative because it is more likely for you to lose than to win.

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Expected value indicates what is likely to happen in the long run, when you play many many times. It may not be representative of a day’s worth of playing. But, the more you play the more likely the expected value represents what happens.
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For example, a casino can estimate fairly well how much money they’ll make, given the number of people playing, by knowing the probability of winning and their payouts. This is because they have a huge number of players.
In general, if you make a bet, then, on average, the amount of money you win (or lose) is given by

\[
\text{Expected Value} = \text{probability of winning} \cdot \text{amount won} - \text{probability of losing} \cdot \text{amount bet}
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A Formula for Expected Value

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For example, in the game mentioned on the last slide, this would give

\[
$3 \cdot (1/5) + (-$1) \cdot 4/5 = -$0.20
\]
Roulette

American roulette is played on a roulette wheel with 38 slots. There are 18 black, 18 red, and 2 green (0 and 00).
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European roulette does not have a 00.
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Let’s see a short clip of roulette playing. I suspect this clip isn’t too representative of what actually happens in a casino.
The Betting Board

There are several different bets. For example,

• Bet on a single number. The payout is $35 for a $1 bet.
• Bet on red or black. The payout is $1 for a $1 bet.
• Bet on even or odd. The payout is $1 for a $1 bet.
• Bet on first, second, or third 12. The payout is $2 for a $1 bet.
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If we bet on a single number, then there is exactly 1 way to win. The probability of winning is then

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If we bet on a single number, then there is exactly 1 way to win. The probability of winning is then \[ \frac{1}{38} \]

The probability of losing is then \[ \frac{37}{38} \]
What is the expected value of betting $1 on a single number? We use the formula for expected value we gave earlier:

\[
\text{Expected Value} = \text{probability of winning} \cdot \text{amount won} - \text{probability of losing} \cdot \text{amount bet}
\]

Our bet is $1 and our payout is $35. The expected value is then

\[
\frac{1}{38} \cdot 35 - \frac{37}{38} \cdot 1 = \frac{35}{38} - \frac{37}{38} = -\frac{2}{38} = -\frac{1}{19}
\]

which is about negative 5¢.
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Our bet is $1 and our payout is $35. The expected value is then

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which is about negative 5¢.
Q What is the expected value of betting on red? Recall the payout for a $1 bet is $1. Don’t enter the negative sign.
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A There are 18 red numbers out of 38, so the probability of winning is \(\frac{18}{38}\) and the probability of losing is \(\frac{20}{38}\). The expected value is then

\[
\frac{18}{38} \cdot \$1 - \frac{20}{38} \cdot \$1 = -\frac{2}{38}
\]

which is the same as for betting on a number.
Q What is the expected value of betting on the first 12? The payout for a $1 bet is $2. Again, don’t enter the negative sign.

There are 12 winning numbers (1-12) and 26 losing numbers. The probability of winning is then \( \frac{12}{38} \) and the probability of losing is \( \frac{26}{38} \). The expected value is then

\[
\frac{12}{38} \cdot 2 - \frac{26}{38} \cdot 1 = \frac{24}{38} - \frac{26}{38} = -\frac{2}{38}.
\]

Again, this is the same as the previous expected values.
Q What is the expected value of betting on the first 12? The payout for a $1 bet is $2. Again, don’t enter the negative sign.

A There are 12 winning numbers (1-12) and 26 losing numbers. The probability of winning is then $12/38$ and the probability of losing is $26/38$. The expected value is then

$$\frac{12}{38} \cdot 2 - \frac{26}{38} \cdot 1 = \frac{24}{38} - \frac{26}{38} = -\frac{2}{38}$$

Again, this is the same as the previous expected values.
Next Time

We will look at the New Mexico Lottery games, compute probabilities of winning and determine the expected value of different games. We’ll see that, even for the same game, not all variants of playing have the same expected value.
You make a $1 bet that you will spin and land on red. The payout for winning is $2. What is the expected value of this bet?

A $-\frac{1}{2}$

B $-\frac{1}{4}$

C $0$

D $\frac{1}{4}$

E $\frac{1}{2}$