The New Mexico Lottery

29 February 2012

Benefitting New Mexico's Future
Today we will discuss the various New Mexico Lottery games and look at odds of winning and the expected value of playing the various games.
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All the roulette games we looked at yesterday have the same expected value. We will see that this is not true for lottery games.
The New Mexico Lottery

We will discuss three of the New Mexico Lottery games.
To play Pick 3 you pick a 3 digit number, anything from 000 to 999. You pay $1 to play.
To play Pick 3 you pick a 3 digit number, anything from 000 to 999. You pay $1 to play.

You then select your play type. Your choices are Straight (exact order), Box (any order), or Straight/Box.

<table>
<thead>
<tr>
<th>Play Type</th>
<th>Example</th>
<th>Your #s</th>
<th>Numbers Drawn</th>
<th>Prize</th>
<th>Approx. Odds 1 in</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Straight (exact order)</strong></td>
<td></td>
<td>123</td>
<td>123</td>
<td>$500</td>
<td>1,000</td>
</tr>
<tr>
<td><strong>Box (any order)</strong></td>
<td></td>
<td>112</td>
<td>112, 121, 211</td>
<td>$160</td>
<td>333.33</td>
</tr>
<tr>
<td>If your numbers consist of 2 of the same number.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Box (any order)</strong></td>
<td></td>
<td>123</td>
<td>123, 132, 213, 231, 312, 321</td>
<td>$80</td>
<td>166.67</td>
</tr>
<tr>
<td>If your numbers consist of 3 different numbers.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Straight/Box (exact or any order)</strong></td>
<td></td>
<td>112</td>
<td>112</td>
<td>(STR) $330**</td>
<td>333.33</td>
</tr>
<tr>
<td>If your numbers consist of 2 of the same number.</td>
<td></td>
<td>112</td>
<td>121, 211</td>
<td>(BOX) $80</td>
<td></td>
</tr>
<tr>
<td><strong>Straight/Box (exact or any order)</strong></td>
<td></td>
<td>123</td>
<td>123</td>
<td>(STR) $290**</td>
<td>166.67</td>
</tr>
<tr>
<td>If your numbers consist of 3 different numbers.</td>
<td></td>
<td>123</td>
<td>132, 213, 231, 312, 321</td>
<td>(BOX) $40</td>
<td></td>
</tr>
</tbody>
</table>
Let’s compute the probability of winning each of the three types of playing. For all types of games, there are 1000 ways to pick a 3-digit number (every number between 0 = 000 to 999). To win playing Straight you need to get the exact winning number. The probability of winning is then \(1/1000\).
Let’s compute the probability of winning each of the three types of playing. For all types of games, there are 1000 ways to pick a 3-digit number (every number between 0 = 000 to 999). To win playing Straight you need to get the exact winning number. The probability of winning is then $1/1000$.

If you play Box, your probability depends on the number you choose. If you pick 3 different digits, you have 6 ways to win. The probability of winning is then $6/1000$, or 1 in 166.67.
Let’s compute the probability of winning each of the three types of playing. For all types of games, there are 1000 ways to pick a 3-digit number (every number between 0 = 000 to 999). To win playing Straight you need to get the exact winning number. The probability of winning is then 1/1000.

If you play Box, your probability depends on the number you choose. If you pick 3 different digits, you have 6 ways to win. The probability of winning is then 6/1000, or 1 in 166.67.

However, if you pick only 2 different digits, you have 3 ways to win, and your probability of winning is 3/1000, or 1 in 333.33.
If you play Straight/Box, then how much you win depends if you hit the exact winning number or a rearrangement of it. But, you will win something when you have the winning digits in some order, so the overall change of winning is the same as for Box.
If you play Straight/Box, then how much you win depends if you hit the exact winning number or a rearrangement of it. But, you will win something when you have the winning digits in some order, so the overall change of winning is the same as for Box.

Say you pick three different digits. You win $290 if you hit the exact winning number, for which you have a \( \frac{1}{1000} \) probability of doing. You win $40 if you hit the other five rearrangements of the winning digits, for which you have a probability of \( \frac{5}{1000} \), or 1 in 200.
Recall that the formula for calculating expected value of a bet is

\[
\text{Expected Value} = \text{probability of winning} \cdot \text{amount won} - \text{probability of losing} \cdot \text{bet amount}
\]
What is the expected value of a Straight bet? You win $500 when you hit the exact winning bet.
Q What is the expected value of a Straight bet? You win $500 when you hit the exact winning bet.

A The probability of winning is $1/1000$, so the probability of losing is $999/1000$. The expected value is

\[
\frac{1}{1000} \cdot $500 - \frac{999}{1000} \cdot $1 = \frac{500}{1000} - \frac{999}{1000} = -\frac{499}{1000}
\]

or just about $-0.50$, about $50\text{¢}$ per $1$ bet.
What is the expected value of a Box bet (with 3 different digits)? You win $80 when you hit any rearrangement of the winning number.

The probability of winning is $\frac{6}{1000}$, so the probability of losing is $\frac{994}{1000}$. The expected value is

$$\frac{6}{1000} \cdot 80 - \frac{994}{1000} \cdot 1 = \frac{480}{1000} - \frac{994}{1000} = -\frac{514}{1000}$$

or about 51¢ per $1 bet. This is slightly worse of a game than the Straight play.
Q What is the expected value of a Box bet (with 3 different digits)? You win $80 when you hit any rearrangement of the winning number.

A The probability of winning is \( \frac{6}{1000} \), so the probability of losing is \( \frac{994}{1000} \). The expected value is

\[
\frac{6}{1000} \cdot $80 - \frac{994}{1000} \cdot $1 = \frac{480}{1000} - \frac{994}{1000} = -\frac{514}{1000}
\]

or about 51¢ per $1 bet. This is slightly worse of a game than the Straight play.
Because the probability of winning is exactly half but the payout is double when you play Box using 2 different digits, the expected value is the same.
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So, for any way of playing Box, the expected value is $-\frac{514}{1000}$. 

Because the probability of winning is exactly half but the payout is double when you play Box using 2 different digits, the expected value is the same.

So, for any way of playing Box, the expected value is $-\frac{514}{1000}$.

This is slightly worse for the player than playing Straight. The difference is not huge.
Here we have a more complicated situation, because there are two ways to win, with different payouts. We need to use a variant of the expected value formula. When you have two ways to win, the expected value is

\[
\text{Probability of winning first way } \times \text{ payout for first way } + \text{ Probability of winning 2nd way } \times \text{ payout for 2nd way } - \text{ Probability of losing } \times \text{ amount bet}
\]
To apply this formula for Straight/Box when we pick 3 different digits, the probability of hitting the exact winning number is \( \frac{1}{1000} \) and the payout is $290. The probability of hitting the right three digits but in the wrong order is \( \frac{5}{1000} \) and the payout is $40. The probability of losing is \( \frac{994}{1000} \).
To apply this formula for Straight/Box when we pick 3 different digits, the probability of hitting the exact winning number is \( \frac{1}{1000} \) and the payout is $290. The probability of hitting the right three digits but in the wrong order is \( \frac{5}{1000} \) and the payout is $40. The probability of losing is \( \frac{994}{1000} \).

The expected value is then

\[
\frac{1}{1000} \cdot $290 + \frac{5}{1000} \cdot $40 - \frac{994}{1000} \cdot $1 = \frac{290}{1000} + \frac{200}{1000} - \frac{994}{1000} = -\frac{504}{1000}
\]

which is about \(-50\)¢ per $1 bet. It is slightly better than Box, slightly worse than Straight.
For all games, the expected value is about $-50¢$ per $1$ bet for players. This means the lottery commission expects to take in about $50¢$ per $1$ played. For example, if, over a period of time, people spend $100$ Million on Pick 3, the lottery would expect to make about $50$ Million after paying off winners.
To play Powerball, you choose 5 numbers from 1 to 59 and a Powerball number from 1 to 35. You pay $2 for each time you play.
To play powerball you choose 5 numbers from 1 to 59 and a Powerball number from 1 to 35. You pay $2 for each time you play.
Twice a week winning numbers are selected. For example, here are the winning numbers for February 25.

![Powerball winning numbers for the 2/25/2012 drawing](image-url)
There are 9 types of wins, depending on how many of the winning numbers you selected, and whether or not you selected the winning Powerball number.
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<table>
<thead>
<tr>
<th>Match</th>
<th>Prize</th>
<th>Odds 1 in</th>
</tr>
</thead>
<tbody>
<tr>
<td>Match ∙∙∙∙∙ ✓</td>
<td>JACKPOT</td>
<td>175,223,510</td>
</tr>
<tr>
<td>Match ∙∙∙∙∙</td>
<td>$1 Million</td>
<td>5,153,633</td>
</tr>
<tr>
<td>Match ∙∙∙∙∙ ✓</td>
<td>$10,000</td>
<td>648,976</td>
</tr>
<tr>
<td>Match ∙∙∙∙</td>
<td>$100</td>
<td>19,088</td>
</tr>
<tr>
<td>Match ∙∙∙∙ ✓</td>
<td>$100</td>
<td>12,245</td>
</tr>
<tr>
<td>Match ∙∙∙</td>
<td>$7</td>
<td>360</td>
</tr>
<tr>
<td>Match ∙∙∙ ✓</td>
<td>$7</td>
<td>706</td>
</tr>
<tr>
<td>Match ∙∙</td>
<td>$4</td>
<td>111</td>
</tr>
<tr>
<td>Match ∙</td>
<td>$4</td>
<td>55</td>
</tr>
</tbody>
</table>

Overall odds of winning a prize are approximately 1 in 31.8
### Powerball 2008 Payouts

<table>
<thead>
<tr>
<th>Match:</th>
<th>Prize:</th>
<th>Approx. odds per $1 play:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Jackpot*</td>
<td>1 in 146,107,962</td>
</tr>
<tr>
<td></td>
<td>$200,000**</td>
<td>1 in 3,563,609</td>
</tr>
<tr>
<td></td>
<td>$10,000**</td>
<td>1 in 584,432</td>
</tr>
<tr>
<td></td>
<td>$100**</td>
<td>1 in 14,254</td>
</tr>
<tr>
<td></td>
<td>$100**</td>
<td>1 in 11,927</td>
</tr>
<tr>
<td></td>
<td>$7**</td>
<td>1 in 291</td>
</tr>
<tr>
<td></td>
<td>$7**</td>
<td>1 in 745</td>
</tr>
<tr>
<td></td>
<td>$4**</td>
<td>1 in 127</td>
</tr>
<tr>
<td></td>
<td>$3**</td>
<td>1 in 69</td>
</tr>
</tbody>
</table>

Overall odds of winning a prize are approximately 1 in 36.6
What does this chart mean?

### Nine Ways to Win!

<table>
<thead>
<tr>
<th>Match</th>
<th>Prize</th>
<th>Odds 1 in</th>
</tr>
</thead>
<tbody>
<tr>
<td>Match</td>
<td>JACKPOT</td>
<td>175,223,510</td>
</tr>
<tr>
<td>Match</td>
<td>$1 Million</td>
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</tr>
<tr>
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<td>$10,000</td>
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<td>$100</td>
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<td>Match</td>
<td>$100</td>
<td>12,245</td>
</tr>
<tr>
<td>Match</td>
<td>$7</td>
<td>360</td>
</tr>
<tr>
<td>Match</td>
<td>$7</td>
<td>706</td>
</tr>
<tr>
<td>Match</td>
<td>$4</td>
<td>111</td>
</tr>
<tr>
<td>Match</td>
<td>$4</td>
<td>55</td>
</tr>
</tbody>
</table>

Overall odds of winning a prize are approximately 1 in 31.8
What does this chart mean?

For example, to win $100, you can either match 4 of the 5 (white) winning numbers but not the Powerball winning number or match 3 of the 5 white winning numbers and match the Powerball winning number.
To play we select 5 out of 59 numbers and 1 out of 35 numbers for the Powerball number.
To play we select 5 out of 59 numbers and 1 out of 35 numbers for the Powerball number.

There are $\binom{59}{5}$ ways to select the 5 white numbers. This is equal to $5,006,386$. There are 35 ways to select the Powerball number.
To play we select 5 out of 59 numbers and 1 out of 35 numbers for the Powerball number.

There are \( \binom{59}{5} \) ways to select the 5 white numbers. This is equal to 5,006,386. There are 35 ways to select the Powerball number.

The total number of ways to play is then

\[
\binom{59}{5} \cdot 35 = 5,006,386 \cdot 35 = 175,223,510
\]
Let’s compute the probability of winning the various ways. First, let’s consider the jackpot.
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To win the jackpot you must match all 5 white winning numbers and the Powerball winning number. There is only 1 way to do this, by picking exactly all the winning numbers.
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To win the jackpot you must match all 5 white winning numbers and the Powerball winning number. There is only 1 way to do this, by picking exactly all the winning numbers.

The probability of winning is then

\[
\frac{1}{175,223,510}
\]
To win $1 Million, we must match all 5 white numbers but not the Powerball number. There is only 1 way to pick all 5 of the white numbers.
To win $1 Million, we must match all 5 white numbers but not the Powerball number. There is only 1 way to pick all 5 of the white numbers.

Q How may ways are there to pick a number that is not the winning Powerball number?
To win $1 Million, we must match all 5 white numbers but not the Powerball number. There is only 1 way to pick all 5 of the white numbers.

Q How may ways are there to pick a number that is not the winning Powerball number?

A There are 34 ways to not pick the Powerball number.
Probability of Winning $1 Million

To win $1 Million, we must match all 5 white numbers but not the Powerball number. There is only 1 way to pick all 5 of the white numbers.

Q How may ways are there to pick a number that is not the winning Powerball number?

A There are 34 ways to not pick the Powerball number.

The probability of winning $1 Million is then

\[
\frac{34}{175,223,510}
\]

which is approximately 1 in 5,153,633 plays.
Here is a table giving the number of ways to win each of the 9 methods of winning.

<table>
<thead>
<tr>
<th>White Balls</th>
<th>Powerball</th>
<th>Formula</th>
<th>Number of Ways</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td></td>
<td>34</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>$\binom{5}{4} \cdot \binom{54}{1}$</td>
<td>270</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>$\binom{5}{4} \cdot \binom{54}{1} \cdot 34$</td>
<td>9,180</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>$\binom{5}{3} \cdot \binom{54}{2}$</td>
<td>14,310</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>$\binom{5}{3} \cdot \binom{54}{2} \cdot 34$</td>
<td>486,540</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>$\binom{5}{2} \cdot \binom{54}{3}$</td>
<td>248,040</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>$\binom{5}{1} \cdot \binom{54}{4}$</td>
<td>1,581,255</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>$\binom{54}{5}$</td>
<td>3,162,510</td>
</tr>
</tbody>
</table>

**Total Number of Ways to Win**  5,502,140
This table shows us that there are 5,502,140 ways to win, and we’ve seen that there are 175,223,510 ways to play.
This table shows us that there are $5,502,140$ ways to win, and we’ve seen that there are $175,223,510$ ways to play.

The probability of winning something is then

$$\frac{5,502,140}{175,223,510}$$

which is about 1 in 31.8.
How can we get this 1 in 31.8 calculation?

If we temporarily write $x$ for the 31.8 number, then $x$ must satisfy the equation:

$$5, 502, 140 = 1, 223, 510$$

Solving for $x$ gives $x = 175, 223, 510$, $5, 502, 140$.

I've used this idea every time I rewrote a probability as 1 in something.
How can we get this 1 in 31.8 calculation?

If we temporarily write \( x \) for the 31.8 number, then \( x \) must satisfy the equation

\[
\frac{5,502,140}{175,223,510} = \frac{1}{x}
\]
How can we get this 1 in 31.8 calculation?

If we temporarily write \( x \) for the 31.8 number, then \( x \) must satisfy the equation

\[
\frac{5,502,140}{175,223,510} = \frac{1}{x}
\]

Solving for \( x \) gives

\[
x = \frac{175,223,510}{5,502,140}
\]

I’ve used this idea every time I rewrote a probability as 1 in something.
Because the jackpot isn’t a prescribed payoff, we can’t determine the expected value of a Powerball play. However, there is one thing we can analyze.
Because the jackpot isn’t a prescribed payoff, we can’t determine the expected value of a Powerball play. However, there is one thing we can analyze.

Powerball gives you the option of paying an extra $1 in return for higher payouts for some of the ways to win.
Assignment 5 is now on the website. It is due one week from today. You have a choice of answering a question related to poker or to lotteries. There are three questions to choose from. You are to answer just one.
Next Time

We’ll look at if paying an extra $1 in Powerball is a good idea. We’ll also look at the Roadrunner Cash game.
You make a $1 bet that you will spin and land on red. The payout for winning is $2. What is the expected value of this bet?

A  $-1/2$
B  $-1/4$
C  0
D  $1/4$
E  $1/2$