The New Mexico Lottery Continued

2 March 2012

Benefitting New Mexico's Future
Last Time

On Wednesday we discussed Pick 3 and Powerball.

Today we’ll finish up our discussion of Powerball and discuss Roadrunner Cash.
To play powerball you choose 5 numbers from 1 to 59 and a Powerball number from 1 to 35. You pay $2 for each time you play.
Twice a week winning numbers are selected. For example, here are the winning numbers for February 25.

![Powerball winning numbers for the 2/25/2012 drawing](image-url)
There are 9 types of wins, depending on how many of the winning numbers you selected, and whether or not you selected the winning Powerball number.
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### Nine Ways to Win!

<table>
<thead>
<tr>
<th>Match</th>
<th>Prize</th>
<th>Odds 1 in</th>
</tr>
</thead>
<tbody>
<tr>
<td>Match ⬤⬤⬤⬤⬤</td>
<td>JACKPOT</td>
<td>175,223,510</td>
</tr>
<tr>
<td>Match ⬤⬤⬤⬤</td>
<td>$1 Million</td>
<td>5,153,633</td>
</tr>
<tr>
<td>Match ⬤⬤⬤</td>
<td>$10,000</td>
<td>648,976</td>
</tr>
<tr>
<td>Match ⬤⬤</td>
<td>$100</td>
<td>19,088</td>
</tr>
<tr>
<td>Match ⬤</td>
<td>$100</td>
<td>12,245</td>
</tr>
<tr>
<td>Match ⬤⬤</td>
<td>$7</td>
<td>360</td>
</tr>
<tr>
<td>Match ⬤⬤</td>
<td>$7</td>
<td>706</td>
</tr>
<tr>
<td>Match ⬤</td>
<td>$4</td>
<td>111</td>
</tr>
<tr>
<td>Match ⬤</td>
<td>$4</td>
<td>55</td>
</tr>
</tbody>
</table>

Overall odds of winning a prize are approximately 1 in 31.8
To play we select 5 out of 59 numbers and 1 out of 35 numbers for the Powerball number.

There are $\binom{59}{5}$ ways to select the 5 white numbers. This is equal to 5,006,386. There are 35 ways to select the Powerball number.

The total number of ways to play is then

$$\binom{59}{5} \cdot 35 = 5,006,386 \cdot 35 = 175,223,510$$
Let’s compute the probability of winning the various ways. First, let’s consider the jackpot.
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To win the jackpot you must match all 5 white winning numbers and the Powerball winning number. There is only 1 way to do this, by picking exactly all the winning numbers.
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To win the jackpot you must match all 5 white winning numbers and the Powerball winning number. There is only 1 way to do this, by picking exactly all the winning numbers.

The probability of winning is then

\[
\frac{1}{175,223,510}
\]
To win $1 Million, we must match all 5 white numbers but not the Powerball number. There is only 1 way to pick all 5 of the white numbers.
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Q How may ways are there to pick a number that is not the winning Powerball number?
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Q How may ways are there to pick a number that is not the winning Powerball number?

A There are 34 ways to not pick the Powerball number.
To win $1 Million, we must match all 5 white numbers but not the Powerball number. There is only 1 way to pick all 5 of the white numbers.

Q How may ways are there to pick a number that is not the winning Powerball number?

A There are 34 ways to not pick the Powerball number.

The probability of winning $1 Million is then

\[
\frac{34}{175,223,510}
\]

which is approximately 1 in 5,153,633 plays.
Here is a table giving the number of ways to win each of the 9 methods of winning.

<table>
<thead>
<tr>
<th>White Balls</th>
<th>Powerball</th>
<th>Formula</th>
<th>Number of Ways</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1</td>
<td>$\binom{5}{5} \cdot \binom{1}{1}$</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>$\binom{5}{5} \cdot \binom{34}{1}$</td>
<td>34</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>$\binom{5}{4} \cdot \binom{54}{1}$</td>
<td>270</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>$\binom{5}{4} \cdot \binom{54}{1} \cdot \binom{34}{1}$</td>
<td>9,180</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>$\binom{5}{3} \cdot \binom{54}{2}$</td>
<td>14,310</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>$\binom{5}{3} \cdot \binom{54}{2} \cdot \binom{34}{1}$</td>
<td>486,540</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>$\binom{5}{2} \cdot \binom{54}{3}$</td>
<td>248,040</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>$\binom{5}{1} \cdot \binom{54}{4}$</td>
<td>1,581,255</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>$\binom{54}{5}$</td>
<td>3,162,510</td>
</tr>
</tbody>
</table>

**Total Number of Ways to Win** 5,502,140
Because the jackpot isn’t a prescribed payoff, we can’t determine the expected value of a Powerball play. However, there is one thing we can analyze.
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Powerball gives you the option of paying an extra $1 in return for higher payouts for some of the ways to win.
Is Increase Your Prizes a Good Idea?

Increase your prizes with Power Play!

For an additional $1 per play per draw, “Power Play” increases your Powerball prize (excluding jackpot). Simply mark the “YES” box for “Power Play” on your play slip or ask your retailer for a Power Play.

<table>
<thead>
<tr>
<th>Match</th>
<th>Prize without Power Play</th>
<th>Prize with Power Play</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>JACKPOT</td>
</tr>
<tr>
<td>Match</td>
<td>$1 Million</td>
<td>$2 Million</td>
</tr>
<tr>
<td>Match</td>
<td>$10,000</td>
<td>$40,000</td>
</tr>
<tr>
<td>Match</td>
<td>$100</td>
<td>$200</td>
</tr>
<tr>
<td>Match</td>
<td>$100</td>
<td>$200</td>
</tr>
<tr>
<td>Match</td>
<td>$7</td>
<td>$14</td>
</tr>
<tr>
<td>Match</td>
<td>$4</td>
<td>$12</td>
</tr>
</tbody>
</table>
The Powerball tickets don’t say what the increased payoff for doing this is. The previous slide was copied from the NM Lottery website.
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We can do a comparison of the expected values of playing regularly or doing the increased payoff. It turns out that the computation doesn’t need to know the jackpot payoff.
Because the expected value formula for this situation is even more complicated, because we’ve got lots of ways to win, we’ll use a symbolic algebra program. In the following slides, $p_1$ through $p_9$ represent the probability of winning for the 9 ways to win, and $q$ is the probability to lose. The letter $j$ represents the jackpot payoff.
Because the expected value formula for this situation is even more complicated, because we’ve got lots of ways to win, we’ll use a symbolic algebra program. In the following slides, \( p_1 \) through \( p_9 \) represent the probability of winning for the 9 ways to win, and \( q \) is the probability to lose. The letter \( j \) represents the jackpot payoff.

The calculation we’ll do will give the expected value \( e_1 \) for playing normally and \( e_2 \) for playing increased payoff as formulas involving \( j \). However, we’ll compute \( e_1 - e_2 \) and see if it is positive or negative. We’ll be able to tell this.
\[ d := 175223510 \]

\[ p1 := \frac{3162510}{d} \]

\[ p2 := \frac{1581255}{d} \]

\[ p3 := \frac{248040}{d} \]

\[ p4 := \frac{486540}{d} \]

\[ p5 := \frac{14310}{d} \]
\[ p_6 := \frac{9180}{d} \]

\[ p_7 := \frac{270}{d} \]

\[ p_8 := \frac{34}{d} \]

\[ p_9 := \frac{1}{d} \]

\[
\begin{align*}
q := 1 - (p_1 + p_2 + p_3 + p_4 + p_5 + p_6 + p_7 + p_8 + p_9) \\
q := \frac{2424591}{2503193}
\end{align*}
\]
\[ \begin{align*}
e1 & := p1 \cdot 4 + p2 \cdot 4 + p3 \cdot 7 + p4 \cdot 7 + p5 \cdot 100 + p6 \cdot 100 + p7 \cdot 10000 + p8 \cdot 1000000 + p9 \cdot j - 2q \\
& \quad - \frac{952678}{604219} + \frac{1}{175223510} j
\end{align*} \]

\[ \begin{align*}
e2 & := p1 \cdot 12 + p2 \cdot 12 + p3 \cdot 14 + p4 \cdot 14 + p5 \cdot 200 + p6 \cdot 200 + p7 \cdot 40000 + p8 \cdot 2000000 + p9 \cdot j - 3q \\
& \quad - \frac{35845681}{17522351} + \frac{1}{175223510} j
\end{align*} \]

\[ e1 - e2 = \frac{8218019}{17522351} \]
Because $e_1 - e_2$ is positive, $e_1 > e_2$. This means the expected value for the usual way to play is larger, so is a better deal. Therefore, even though the payoffs are bigger doing increased payoff, the main beneficiary is the lottery, not the players.
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In fact, $e_1 - e_2$ is about 50¢. This means, by playing increased payout, on average you lose an extra 50¢ per $1 bet every time you play!
We can use these calculations to answer some questions about expected value.
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For example, what should the jackpot value be if the lottery wants the expected value to be $-1$ per $2$ bet?
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We are asking, for what value of $j$, is

$$-\frac{952678}{604219} + \frac{j}{175223510} = -1$$
To solve for $j$ we can move the first fraction to the right by adding it to both sides, then multiplying by $175,223,510$. The result is

$$j = 175,223,510 \cdot \left( \frac{952678}{604219} - 1 \right)$$

$$= \$101,053,110$$
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If the lottery did this and made the expected value $-\$1$ per $2$ bet for players, then they would expect to make $\$1$ per $2$ bet.
To solve for $j$ we can move the first fraction to the right by adding it to both sides, then multiplying by $175,223,510$. The result is

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$$= 101,053,110$$

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This represents the long run average. Whenever somebody wins the jackpot, that changes dramatically the amount of money the lottery makes. An expected value of $-\$1$ per $\$2$ bet means the lottery expects to make half as much as is played. For example, they’d expect to make $500,000 for each $1$ Million played, on average.
<table>
<thead>
<tr>
<th>PLAY A $1</th>
<th>PLAY B $1</th>
<th>PLAY C $1</th>
<th>PLAY D $1</th>
<th>PLAY E $1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 2 3 4 5 6 7 8</td>
<td>1 2 3 4 5 6 7 8</td>
<td>1 2 3 4 5 6 7 8</td>
<td>1 2 3 4 5 6 7 8</td>
<td>1 2 3 4 5 6 7 8</td>
</tr>
<tr>
<td>17 18 19 20 21 22 23 24</td>
<td>17 18 19 20 21 22 23 24</td>
<td>17 18 19 20 21 22 23 24</td>
<td>17 18 19 20 21 22 23 24</td>
<td>17 18 19 20 21 22 23 24</td>
</tr>
<tr>
<td>33 34 35 36 37</td>
<td>33 34 35 36 37</td>
<td>33 34 35 36 37</td>
<td>33 34 35 36 37</td>
<td>33 34 35 36 37</td>
</tr>
</tbody>
</table>

Pick 5 numbers

QP VOID

QP VOID

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This game is a simpler version of Powerball. In Roadrunner cash you pick 5 numbers out of 37. You pay $1 to play.
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Five winning numbers are selected.
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Five winning numbers are selected.

Winning numbers are shown on the lottery website. Here are the winning numbers from February 27.

Did You Win?

Roadrunner Cash winning numbers for the 2/27/2012 drawing

17 26 27 33 37

Click here for previous drawing results
You win if you picked at least 2 of the winning numbers. The more winning numbers you pick the more money you win.
You win if you picked at least 2 of the winning numbers. The more winning numbers you pick the more money you win.

**How to Win**

Five numbers, each from a range of 1 to 37 will be drawn nightly. You win the prize indicated by matching the numbers drawn, as shown in the following prize chart:

<table>
<thead>
<tr>
<th>Match</th>
<th>Win</th>
<th>Approximate Odds per $1 play</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 of 5</td>
<td>Jackpot</td>
<td>1 in 435,897</td>
</tr>
<tr>
<td>4 of 5</td>
<td>$200</td>
<td>1 in 2,724.36</td>
</tr>
<tr>
<td>3 of 5</td>
<td>$5</td>
<td>1 in 87.88</td>
</tr>
<tr>
<td>2 of 5</td>
<td>$1</td>
<td>1 in 8.79</td>
</tr>
</tbody>
</table>

Overall odds of winning a prize are approximately 1 in 7.97
To play you select 5 numbers out of 37. There are \( \binom{37}{5} \) ways to do this, which is equal to 435,897.
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To win the top prize, you must pick all 5 winning numbers. Since there is only 1 way to do this, the probability of winning the top prize is

$$\frac{1}{435,897}$$
To win $200 you must pick exactly 4 of the 5 winning numbers. You then also pick 1 of the 32 losing numbers.
To win $200 you must pick exactly 4 of the 5 winning numbers. You then also pick 1 of the 32 losing numbers.

There are $\binom{5}{4} = 5$ ways to pick 4 of the 5 winning numbers. There are $\binom{32}{1} = 32$ ways to pick 1 of the losing numbers. All together, there are $5 \cdot 32 = 160$ ways to win in this way.
To win $200 you must pick exactly 4 of the 5 winning numbers. You then also pick 1 of the 32 losing numbers.

There are $5 \binom{4}{5} = 5$ ways to pick 4 of the 5 winning numbers. There are $32 \binom{1}{32} = 32$ ways to pick 1 of the losing numbers. All together, there are $5 \cdot 32 = 160$ ways to win in this way.

The probability of winning $200 is then

\[
\frac{160}{435,897}
\]

which is about 1 out of 2,724.
If we figure out the number of ways to win for each way, we’ll get the following table.
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<table>
<thead>
<tr>
<th>Match to Win</th>
<th>Formula</th>
<th>Number of Ways</th>
<th>Prize</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 of 5</td>
<td>(_5C_5)</td>
<td>1</td>
<td>Top Prize</td>
</tr>
<tr>
<td>4 of 5</td>
<td>(_5C_4 \cdot _{32}C_1)</td>
<td>160</td>
<td>$200</td>
</tr>
<tr>
<td>3 of 5</td>
<td>(_5C_3 \cdot _{32}C_2)</td>
<td>4960</td>
<td>$5</td>
</tr>
<tr>
<td>2 of 5</td>
<td>(_5C_2 \cdot _{32}C_3)</td>
<td>49600</td>
<td>$1</td>
</tr>
<tr>
<td><strong>Total Number of Ways to Win</strong></td>
<td></td>
<td><strong>54,721</strong></td>
<td></td>
</tr>
</tbody>
</table>
If we figure out the number of ways to win for each way, we’ll get the following table.

<table>
<thead>
<tr>
<th>Match to Win</th>
<th>Formula</th>
<th>Number of Ways</th>
<th>Prize</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 of 5</td>
<td>$5C_5$</td>
<td>1</td>
<td>Top Prize</td>
</tr>
<tr>
<td>4 of 5</td>
<td>$5C_4 \cdot 32C_1$</td>
<td>160</td>
<td>$200</td>
</tr>
<tr>
<td>3 of 5</td>
<td>$5C_3 \cdot 32C_2$</td>
<td>4960</td>
<td>$5</td>
</tr>
<tr>
<td>2 of 5</td>
<td>$5C_2 \cdot 32C_3$</td>
<td>49600</td>
<td>$1</td>
</tr>
</tbody>
</table>

**Total Number of Ways to Win** 54,721

The overall probability of winning something is then $\frac{54,721}{435,897}$, which is about 1 in 8.
Next Week

We will discuss mathematical aspects of voting, including different voting systems and how different systems can affect who gets elected. We’ll also look at some of the more interesting elections involving 3 (or more) candidates and how the presence of a third party candidate affected the results.
Suppose playing the New Mexico Lottery has an overall expected value of $-50\cent$ per $1$ play. Suppose that, over some time period, people spend $100$ Million playing the lottery. Approximately how much revenue will the lottery make after paying out prizes.

A $100$ Million
B $50$ Million
C Nothing
D The lottery organization will lose money