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To understand the method we needed the idea of proportionality, and proportions we get from similar triangles.
Recall that two triangles are similar if their corresponding angles are equal.
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When this happens, corresponding sides are proportional. This means that the ratio of a side in the big triangle to the corresponding side of the small triangle is the same for each of the three pairs of sides.
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In equation form, we have

\[
\frac{EG}{AB} = \frac{EF}{AC} = \frac{GF}{BC}
\]
Using Thales’ method of shadows, suppose the measurements are given above. How tall is the Eiffel Tower?
We have the ratio

\[
\frac{\text{height of the tower}}{10} = \frac{500}{5} = 100
\]

Multiplying by 10 gives the height to be \(10 \cdot 100 = 1000\) feet.
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We’ll now look at some related methods.
The Eye Sighting Method

To measure the height of the tree, have somebody stand in front of it. You then lay on the ground far enough from your friend so that you line up the top of her head with the top of the tree.
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We have similar triangles; the angles of the two triangles are marked. Both share the green-marked angle. The red-marked angles are right angles; that is, they measure $90^\circ$. 
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We have similar triangles; the angles of the two triangles are marked. Both share the green-marked angle. The red-marked angles are right angles; that is, they measure \(90^\circ\).

There are a couple ways to see the blue-marked angles are equal. One way is that the sum of the angles of a triangle is \(180^\circ\). This means each of the blue-marked angles is

\[
180^\circ - \text{red-marked angle} - \text{green-marked angle}
\]
Because we have similar triangles, corresponding sides are proportional. That is, the ratio of a side of the big triangle to the corresponding side of the smaller triangle is the same for each of the three pairs of sides.

We only need to compare two pairs of sides. Doing so, we have:

\[
\frac{\text{height of tree}}{\text{height of girl}} = \frac{\text{distance to tree}}{\text{distance to girl}}
\]
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Looking at our equation

\[
\frac{\text{height of tree}}{\text{height of girl}} = \frac{\text{distance to tree}}{\text{distance to girl}}
\]

to calculate the height of the tree we can measure the other three values: The height of the girl, the distance to the girl and the distance to the tree.
Suppose the girl is 4 feet tall, the distance to the girl is 10 feet, and the distance to the tree is 50 feet. What is the height of the tree?

\[
\frac{\text{height of tree}}{\text{height of girl}} = \frac{\text{distance to tree}}{\text{distance to girl}}
\]
If we plug the known values into the equation

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\frac{\text{height of tree}}{4} = \frac{50}{10} = 5
\]
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\frac{\text{height of tree}}{\text{height of girl}} = \frac{\text{distance to tree}}{\text{distance to girl}}
\]

we get

\[
\frac{\text{height of tree}}{4} = \frac{50}{10} = 5
\]

Multiplying both sides by 4 gives

\[
\text{height of tree} = 4 \cdot 5 = 20 \text{ feet}
\]
A Variation to Measure the Width of a River

To find the width of the river we need to measure the distance between two points on either bank of the river. We also need to measure the distance to the near bank.

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With a little algebra we can find the width of the river.
Using similar triangles, we get

\[
\frac{120 + \text{width of the river}}{120} = \frac{80}{2} = 2
\]

We can rewrite this as

\[
120 + \text{width of the river} = 120 \cdot 2 = 240
\]

giving the width to be 120 feet.
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giving the width to be 120 feet.
One drawback to the eye sighting method is it requires us to be able to measure the distance to the tree. This may not always be possible. For example, if the tree is across a river, we might not be able to measure the distance to the tree.
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It also takes careful use to get decent results. Having the distance to the girl be a little off can make a big difference in the height of the tree, if the tree is moderately far away.
The Sea Island Method

This method was described in a manuscript written by the Chinese mathematician Liu Hui in the 3rd century. It describes measuring the height of a mountain on an island. While using similar triangles is the key, the use is more sophisticated and gives us more information. The method allows us to determine the height of the mountain and the distance to the mountain.
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While using similar triangles is the key, the use is more sophisticated and gives us more information. The method allows us to determine the height of the mountain and the distance to the mountain.
The Sea Island method uses the eye sighting method twice. We place two sticks in the ground and line up the top of each stick with the mountain. Besides the height of the stick, there are three distances we need to calculate.
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We've got two pairs of similar triangles. As with the eye sighting method, we will get an equation for each pair of similar triangles.

$$a = \text{distance from eye to first stick}$$

$$b = \text{distance from eye to second stick}$$

$$h = \text{height of stick}$$
We've got two pair of similar triangles. As with the eye sighting method, we will get an equation for each pair of similar triangles.
\[
\frac{\text{height of the mountain}}{h} = \frac{D + a}{a}
\]
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\]
An Example

We will illustrate the method through a specific example.
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Let’s abbreviate the height of the mountain as $H$. Our two equations are then

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\[ \frac{D + 10}{10} = \frac{D + 110.1}{10.1} \]

If we cross multiply, we get

\[ 10.1(D + 10) = 10(D + 110.1) \]
From

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if we multiply out we get

\[ 10.1D + 101 = 10D + 1101 \]
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Subtracting 101 from both sides gives \( 10.1D = 10D + 1000 \), then subtracting \( 10D \) gives \( .1D = 1000 \).
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if we multiply out we get

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Subtracting 101 from both sides gives \(10.1D = 10D + 1000\), then subtracting \(10D\) gives \(.1D = 1000\).

Multiplying by 10 (which is the same as dividing by .1) yields

\[D = 10 \cdot 1000 = 10,000 \text{ feet}\]
Our first equation a couple sides ago was

\[
\frac{H}{3} = \frac{D + 10}{10}
\]

which with \( D = 10,000 \) simplifies to

\[
\frac{H}{3} = \frac{10,010}{10} = 1001
\]
Our first equation a couple sides ago was

\[ \frac{H}{3} = \frac{D + 10}{10} \]

which with \( D = 10,000 \) simplifies to

\[ \frac{H}{3} = \frac{10,010}{10} = 1001 \]

Multiplying by 3 gives

\[ H = 3 \cdot 1001 = 3,003 \text{ feet} \]
Next Time

We will see how an extension of these ideas will help us determine the size of the earth and the distance to the sun and moon.
True or False: The Sea Island Method can give more information than the other methods we’ve studied.

A  True
B  False