Interest Rates: Inflation and Loans

25 April 2012
On Monday we discussed compound interest and saw that money can grow very large given enough time, or a high enough interest rate. We’ll see how this is relevant for discussing loans. Home loans, which often run for 30 years, are over a long enough period of time that rates of interest are very significant. We’ll see why.
On Monday we discussed compound interest and saw that money can grow very large given enough time, or a high enough interest rate. We’ll see how this is relevant for discussing loans. Home loans, which often run for 30 years, are over a long enough period of time that rates of interest are very significant. We’ll see why.

The last assignment of the semester (# 9) is now on the course website. It is due Friday 4 May in class. You need to do one problem from the six choices given.
Clicker Question

My grandparent’s bought a house in San Francisco for $6,000. If I offered them $12,000 for the house, which response below do you think would most represent their feelings about the offer?

A Very happy
B Satisfied
C Unhappy
D Disgusted
E Would call the police on me
While I don’t know if they’d call the cops, they certainly would not be interested in the offer.
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Why not?
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Why not?

Because inflation affects house prices.
The mathematics of inflation on the value of money is similar to that of compound interest.
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If the inflation rate averages 5% per year, an item that cost $100 today would be expected to cost $100 \cdot 1.05 = $105 in one year.
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If the inflation rate averages 5% per year, an item that cost $100 today would be expected to cost \( \$100 \cdot 1.05 = \$105 \) in one year.

How much would it be expected to cost in 2 years?
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If the inflation rate averages 5% per year, an item that cost $100 today would be expected to cost $100 \cdot 1.05 = $105 in one year.

How much would it be expected to cost in 2 years?

It would be 105% of $105, or

$$105 \cdot 1.05 = 100 \cdot 1.05 \cdot 1.05 = 100 \cdot 1.05^2$$

which is $110.25.
If $P$ is the present value of money, at an inflation rate of $r\%$ per year (made into a decimal), if $F$ is the equivalent value $n$ years later, then $F$ satisfies the formula

$$F = P \cdot (1 + r)^n$$
A Formula for Inflation

If $P$ is the present value of money, at an inflation rate of $r\%$ per year (made into a decimal), if $F$ is the equivalent value $n$ years later, then $F$ satisfies the formula

$$F = P \cdot (1 + r)^n$$

This is exactly the same formula as for compound interest. So inflation behaves very much like compound interest. Having a high inflation rate year after year really erodes the value of money.
If you earn $50,000 a year today, what will you have to earn in 20 years in order to have the same income level, if inflation were 2% per year?

Ans At 2% per year, the equivalent value in 20 years of $50,000 is $50,000 \times (1 + 0.02)^{20} = $74,297. If the inflation rate was 5% per year, then the equivalent value in 20 years is $50,000 \times (1 + 0.05)^{20} = $132,665.
Clicker Question

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The calculation just done is an example of what is called a future value calculation. Because it uses the same formula as the compound interest formula, it is computed in exactly the same way as a compound interest question.
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Asking the question in reverse is an a present value calculation. If

\[ F = P \cdot (1 + r)^n \]

is the future value formula, we can solve for \( P \) by dividing by \((1 + r)^n\). Doing so gives

\[ P = \frac{F}{(1 + r)^n} = F \cdot (1 + r)^{-n} \]
If you think you will need $500,000 for retirement in 30 years, and inflation is 2% per year, how much would this amount be worth now?

Ans: It would be worth $276,035.44.

If you want to enter this on a typical scientific calculator, the steps are:

500000 \times \left(1 + 0.02\right)^{30} =

or

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You don't hit the minus sign to enter the −30. You hit the negation sign or the ± sign, depending on the calculator.
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If you borrow money, how is the monthly payment determined?
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What does it even mean to say you get a car loan at an annual interest rate of 6%?
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2. The company gives you the loan, and then invests each payment you make at 6% per year, compounded monthly.
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2. The company gives you the loan, and then invests each payment you make at 6% per year, compounded monthly.

To say you have a 6% loan for 5 years means the loan company would have the same amount of money in each of the two scenarios.
If the company invests each payment $P$ at an annual interest rate $r$, then the last payment does not generate interest, so is worth exactly $P$. To simplify writing we’ll abbreviate $r/12$ by $q$. This is the monthly interest rate.
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And so on. If you have $n$ payments, then the first payment generates $n - 1$ months interest, so is worth $P \cdot (1 + q)^{n-1}$ at the end.
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If $L$ is the loan amount, if the company invested the money instead of giving it to you, after $n$ months it would have $L \cdot (1 + q)^n$. 
So, the payment satisfies the equation

\[ L \cdot (1 + q)^n = P + P \cdot (1 + q) + \cdots + P \cdot (1 + q)^{n-1} \]

\[ \quad = P \cdot (1 + (1 + q) + \cdots + (1 + q)^{n-1}) \]

since both sides represent how much money the loan company would have at the end of your loan in the two different scenarios we mentioned above.
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since both sides represent how much money the loan company would have at the end of your loan in the two different scenarios we mentioned above.

Fortunately, expressions like the one on the right occur often, and people have found formulas to simplify them.
For example, suppose we consider the expression

\[ s = 1 + 3 + 3^2 + \cdots + 3^{n-1} \]

Then

\[ s + 3^n = 1 + 3 + \cdots + 3^{n-1} + 3^n = 1 + 3 \cdot (1 + 3 + \cdots + 3^{n-1}) = 1 + 3s \]

Rearranging gives \( 3^n - 1 = 2s \), and so

\[ s = \frac{3^n - 1}{2} \]
More generally, if $a$ is any number (other than 1), then

$$1 + a + a^2 + \cdots + a^{n-1} = \frac{a^n - 1}{a - 1}$$
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Applying this to the loan situation, with $a = 1 + q$ gives us

$$\left(1 + (1 + q) + \cdots + (1 + q)^{n-1}\right) = \frac{(1 + q)^n - 1}{q}$$
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Thus, our loan formula simplifies as

$$L \cdot (1 + q)^n = P \cdot \left(1 + (1 + q) + \cdots + (1 + q)^{n-1}\right) = P \cdot \left(\frac{(1 + q)^n - 1}{q}\right)$$
Solving

\[ L \cdot (1 + q)^n = P \cdot \left( \frac{(1 + q)^n - 1}{q} \right) \]

for \( P \) gives

\[ P = \frac{Lq \cdot (1 + q)^n}{(1 + q)^n - 1} \]

or, if we divide the top and bottom of the fraction by \((1 + q)^n\),

\[ P = \frac{Lq}{1 - (1 + q)^{-n}} \]
Loan Formula

To summarize, if we borrow $L$ at an interest rate of $r$ per year, and make $n$ payments, then the monthly payment $P$ is

$$P = \frac{Lr}{12 \left( 1 - (1 + \frac{r}{12})^{-n} \right)}$$
To summarize, if we borrow $L$ at an interest rate of $r$ per year, and make $n$ payments, then the monthly payment $P$ is

$$P = \frac{Lr}{12 \left(1 - (1 + \frac{r}{12})^{-n}\right)}$$

Using the Interest Calculator spreadsheet or an online financial calculator is a good way to do these calculations. Moneychimp.com has a simple to use calculator.
Clicker Questions

1. What is the monthly payment for a $20,000 car loan at an annual interest rate of 6% for 5 years?

The loan formula is

\[ P = \frac{Lr}{12 \left(1 - \left(1 + \frac{r}{12}\right)^{-n}\right)} \]

To enter the loan formula on a calculator, enter the following key strokes

\[ L \times r / \left(12 \times \left(1 - \left(1 + \frac{r}{12}\right)^{-n}\right)\right) = \]
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3 A Mercedes S65 AMG starts at $211,000. If you borrowed $200,000 at 6% for 5 years to buy one, what would your monthly payment be?
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Ans It would be 10 times the first answer, or $3,866.56.
If you bought the Mercedes, your total payments for the loan would be

$$3,866.56 \cdot 60 = 231,993.60$$
If you bought the Mercedes, your total payments for the loan would be

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This means you’d pay about $32,000 in interest in order to borrow $200,000 for 5 years at 6%.
With home loans typically for 30 years, interest is much more an issue than for car loans.
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Getting a 15 year loan can save a huge amount of money!
Let’s suppose you borrow $150,000 to buy a house. Let’s consider various options, starting with 30 year loans.

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How much would your monthly payment be if your interest rate was 4%? What about 6%? What about 10%? Finally, what about 15%? We’ll use the Interest Calculator spreadsheet to do this. The results do not include real estate taxes and insurance, which can be a few hundred dollars a month.
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If you get a 15 year home loan, generally you’ll get a better interest rate. Let’s compare a 30 year loan at 10% to a 15 year loan at 9%. We’ll continue to consider a $150,000 loan. Again, we’ll use the Interest Calculator spreadsheet for this.
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While affording the higher monthly payment may not always be possible, if you can do it you’ll save a lot of money in the long run.
While this isn’t as relevant now since interest rates are so low, at times interest rates drop a few points after you buy a house.
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If you borrow $150,000 for 30 years at 7%, and you have the opportunity to refinance with a 15 year loan at 4%, is it a good idea? At least, what are the monthly payments?

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Next time we’ll discuss credit cards and begin talking about annuities.
You can save a lot of money in the long run by getting a lower interest rate on a mortgage loan.

A  True

B  False