Last time we discussed loans and saw how big an effect interest rates were on a loan, especially a home loan, due to the long time periods involved. Today we’ll discuss credit cards. Interest is a big issue for credit cards due to the typically high interest rates charged.
Credit cards work the same as loans. The main difference is the high interest rate most charge. Interest rates up to 20% per year have been common.
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If you pay off your credit card in full each month, then you don’t get charged interest. Using a credit card this way amounts to treating it as a debit card from your checking account.
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If you pay off your credit card in full each month, then you don’t get charged interest. Using a credit card this way amounts to treating it as a debit card from your checking account.

What happens if you don’t pay it off in full? More particularly, what if you pay the minimum payment each month?
The minimum payment on a credit card bill is typically the larger of a fixed amount and a certain percentage of your balance.
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Suppose your minimum payment is the larger of $20 or 1.5% of your balance. Let’s suppose the credit card company charges you 15% interest on unpaid balances. Let’s also suppose you have a $10,000 credit limit, and you max out your credit card.
The minimum payment on a credit card bill is typically the larger of a fixed amount and a certain percentage of your balance.

Suppose your minimum payment is the larger of $20 or 1.5% of your balance. Let’s suppose the credit card company charges you 15% interest on unpaid balances. Let’s also suppose you have a $10,000 credit limit, and you max out your credit card.

Your next statement then shows a $10,000 balance.
What is your minimum payment on a $10,000 balance, when the credit card company requires you to pay at least the larger of $20 or 1.5% of your balance?
Clicker Question

What is your minimum payment on a $10,000 balance, when the credit card company requires you to pay at least the larger of $20 or 1.5% of your balance?

**Ans** Since 1.5% of $10,000 is

\[ 10,000 \cdot 0.015 = 150 \]

your minimum payment is $150.
Let’s now think what will happen if you make the minimum payment each month on your credit card. Your credit card company will charge you $15%/12 \approx 1.25\%$ interest on an unpaid balance each month.
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You pay your minimum payment of $150 the first month. The next statement you’ll have a starting balance of $9,850 since you paid $150 of your $10,000 balance. The company will then charge you 1.25% of that in interest.
1. If your balance is $9,850 and you pay 1.25% interest, how much interest do you owe that month?

Answer: You owe $9,850 \times 0.0125 = $123 in interest charges that month.

2. What will be your next monthly payment, if you pay the minimum payment?

Answer: Your new balance is $9,850 + $123 = $9,973, and you will have to pay 1.5% of that for your minimum payment. So, your minimum payment is $9,973 \times 0.015 = $149.50.
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Interest Rates: Credit Cards and Annuities 27 April 2012 7/25
Clicker Questions

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We can continue seeing what happens when we pay the minimum payment each month. The result for the first year will be the following table.
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<table>
<thead>
<tr>
<th>Month</th>
<th>Original Balance</th>
<th>Interest Charged</th>
<th>New Balance</th>
<th>Payment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$10,000.00</td>
<td></td>
<td>$10,000.00</td>
<td>$150.00</td>
</tr>
<tr>
<td>2</td>
<td>$9,850.00</td>
<td>$123.13</td>
<td>$9,973.13</td>
<td>$149.60</td>
</tr>
<tr>
<td>3</td>
<td>$9,823.53</td>
<td>$122.79</td>
<td>$9,946.32</td>
<td>$149.19</td>
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<tr>
<td>4</td>
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<td>$9,919.59</td>
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<tr>
<td>5</td>
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<td>$122.13</td>
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<td>$148.39</td>
</tr>
<tr>
<td>6</td>
<td>$9,744.54</td>
<td>$121.81</td>
<td>$9,866.35</td>
<td>$148.00</td>
</tr>
<tr>
<td>7</td>
<td>$9,718.35</td>
<td>$121.48</td>
<td>$9,839.83</td>
<td>$147.60</td>
</tr>
<tr>
<td>8</td>
<td>$9,692.23</td>
<td>$121.15</td>
<td>$9,813.38</td>
<td>$147.20</td>
</tr>
<tr>
<td>9</td>
<td>$9,666.18</td>
<td>$120.83</td>
<td>$9,787.01</td>
<td>$146.81</td>
</tr>
<tr>
<td>10</td>
<td>$9,640.21</td>
<td>$120.50</td>
<td>$9,760.71</td>
<td>$146.41</td>
</tr>
<tr>
<td>11</td>
<td>$9,614.30</td>
<td>$120.18</td>
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<td>12</td>
<td>$9,588.46</td>
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<td>$9,708.32</td>
<td>$145.62</td>
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</table>

Totals | $1,336.32         |                  | $1,773.63     |
After 12 months of payments you’ll have paid around $1,774 with $1,336 of that in interest. You will still owe nearly $9,600.
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One problem with this, besides the large amount of interest you pay, is that you won’t be able to use your credit card very much, because your balance is close to your credit limit.
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One problem with this, besides the large amount of interest you pay, is that you won’t be able to use your credit card very much, because your balance is close to your credit limit.

To continue, we could keep going month by month. However, with some algebra we can be more efficient. This will help to see what will happen after several years.
Suppose the original balance in some month (after the first) is $B$. We then get charged $B \cdot 0.0125$ interest. The new balance is then

$$B + B \cdot 0.0125 = B \cdot 1.0125$$
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Our minimum payment is 1.5% of this, or \( B \cdot 1.0125 \cdot 0.015 \). Our next month’s balance is then the previous month’s new balance minus the minimum payment.
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That is, the next month’s original balance is

$$\text{new balance} - \text{payment} =$$

$$= B \cdot 1.0125 - B \cdot 1.0125 \cdot 0.015$$

$$= B \cdot 1.0125 \cdot 0.985$$

$$= B \cdot 0.9973$$
Using the same reasoning, the original balance two month's later is

\[(B \cdot 0.9973) \cdot 0.9973 = B \cdot (0.9973)^2\]
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In general, \( n \) months later, the original balance would be

\[ B \cdot (0.9973)^n \]
This helps to figure out interest and payments years at a time. The results are summarized in the spreadsheet *Credit Card Calculations.xlsx*. 
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<td>$6,763.46</td>
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<td>$7,429.69</td>
<td>59</td>
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<td>$8,322.16</td>
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These calculations are the reasons credit card companies are happy for people to pay the minimum payment.
Annuities

An annuity is a repeating payment, typically of a fixed amount, over a period of time. An annuity is like a loan in reverse; rather than paying a loan company, a bank or investment company pays you the monthly payment.
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1. Paying into an annuity
2. Collecting from an annuity
Paying into an annuity means investing or saving money in order to receive an annuity in the future.
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Collecting from annuity then happens once you have invested enough money to receive an annuity.
Annuities are similar to loans, but with you and the loan company switching roles.
Paying into an Annuity

Annuities are similar to loans, but with you and the loan company switching roles.

If you invest $P$ each month into an account paying an annual interest rate $r$, compounded monthly, and you do this for $n$ months, then the amount of money you’ll have in the bank at the end of the $n$ deposits can be found from the work we did on loans:

$$F = 12P \left( \frac{(1 + \frac{r}{12})^n - 1}{r} \right)$$
If we know how much we want in the future and want to calculate how much we need to invest each month, we can solve the previous formula for $P$, the monthly investment. If $F$ is the future amount, then

$$P = F \left(1 + \frac{r}{12}\right)^n - 1$$
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$$P = \frac{Fr}{(1 + \frac{r}{12})^n - 1}$$

The main difficulty with this calculation is the assumption that you can get a constant rate of return. In practice this isn’t true. These kind of calculations are then approximate at best.
Suppose you need to have $100,000 saved 20 years from now. If you can invest at 6% per year, how much do you need to put away each month?

Recall the relevant formula is

\[ P = \frac{Fr}{(1 + \frac{r}{12})^n - 1} \]
We’d have to invest $216.43 each month to end up with $100,000 in 20 years if we received 6% on our money.
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We can calculate this with the Interest Calculator spreadsheet or with web calculators.
If you are to be paid an monthly annuity at an annual interest rate of $r$ (converted to a decimal) for $m$ years, and you’ve invested $L$ dollars, your monthly income $P$ will be

$$P = \frac{Lr}{12 \left(1 - (1 + \frac{r}{12})^{-n}\right)}$$
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$$P = \frac{Lr}{12 \left(1 - \left(1 + \frac{r}{12}\right)^{-n}\right)}$$

This is the same formula as for loans, because an annuity is really a loan in which you are the lender.
1 Suppose you’ve saved $250,000 that you put into an annuity paying 5% per year, and you wish to collect from it for 20 years. How much monthly income will you receive?

Answer: You’ll receive $1,649.89. If you do this sort of calculation 30 years before you plan to retire, you need to realize the amount you’ll receive will be in future dollars and will sound better than the same amount today.

2 What if you want to collect for 30 years?

Answer: You will collect $1,342.05 each month for 30 years.
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Let’s look at some data, where we take a larger and larger number of years to collect from the annuity.
If you invest $250,000 in an annuity paying 5% per year, this table shows your monthly income depending on how long you collect.

<table>
<thead>
<tr>
<th>Number of Years to Collect</th>
<th>Monthly Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>$1,342</td>
</tr>
<tr>
<td>40</td>
<td>$1,205</td>
</tr>
<tr>
<td>50</td>
<td>$1,135</td>
</tr>
<tr>
<td>60</td>
<td>$1,097</td>
</tr>
<tr>
<td>80</td>
<td>$1,061</td>
</tr>
<tr>
<td>100</td>
<td>$1,049</td>
</tr>
<tr>
<td>150</td>
<td>$1,042</td>
</tr>
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</table>

In fact, you can collect a reasonable amount indefinitely.
If you have $L$ invested in an annuity paying an annual rate of $r$, then you’ll collect

$$\frac{L \cdot r}{12}$$

each month from a perpetual annuity.
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For example, in the previous example, $L = $250,000 and $r = 0.05$. So, the monthly return would be

$$\frac{$250,000 \cdot 0.05}{12} = $1,041.67$$
Next Week

We will investigate the ideas of exponential growth and decay. The mathematics is the same as that of compound interest. Some of the applications we’ll consider are population growth, radioactive decay, determining the time of death, and why you can zoom in with a Google map so quickly.
Paying the just minimum payment on a credit card balance over and over is a good idea financially.

A  True

B  False