Texas Hold’em: How Probability Can Affect Strategy

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Let’s start with a youtube video of Homer and Bart playing Texas Hold’em.

Simpson’s Video
How is Texas Hold’em Played

- Two cards are dealt to each player. A round of betting ensues.

- Three cards are then dealt face up (the flop). These are community cards; anybody can use them. Another round of betting is done.

- Another card is dealt face up (the turn). A round of betting ensues.

- A final card is dealt face up (the river). The final round of betting occurs.

- Of the players that have not folded, the player with the best 5-card hand, made from any combination of his two cards and the cards on the board, wins.
You start Hold’em by being dealt a 2 card hand. How many different 2 card hands are there?

Since you get 2 cards from the 52 card deck, then number is \( \binom{52}{2} \), which is equal to 1,326.

We’ll use this to compute the probability of being dealt a pair with your two cards and, if time permits, the probability of getting 2 cards of the same suit.
Q How many ways are there to deal the 5 cards face up? It involves a binomial coefficient \( nC_r \). The value of \( r \) is 5. What is the value of \( n \)?

A Since you are seeing 2 of the cards, there is only 50 cards to choose from, so the number of ways is \( 50C_5 \), which is equal to 2,118,760.

If we are only interested in how many ways we can flop 3 cards, the number is \( 50C_3 = 19,600 \).

We’ll use this to calculate the probability of ending up with a given hand in Hold’em.
What is the probability you are dealt a pair? This is called a pocket pair.

In order to write down all possible pairs, we need to choose the value of the pair, then choose the actual two cards.

There are 13 choices for the value of the pair.

Q Once we pick the value, how many choices are there for the two cards in the pair?

A There are $4C_2 = 6$ choices for the two cards in the pair.
The total number of ways to be dealt a pair is then $13 \cdot 6 = 78$.

Recall that the total number of 2-card hands you can be dealt is $\binom{52}{2} = 1326$.

Then the probability of being dealt a pocket pair is $\frac{78}{1326}$, or 1 out of 17 hands.
If you are interested in flushes, you’d like to know how likely it is that you are dealt two cards of the same suit to begin the game.

To be dealt 2 of the same suit, you much choose the suit and then choose the cards of that suit.

There are 4 choices for the suit.

Q  How many choices are there to pick the 2 cards of the suit? It is $nC_2$ for some $n$. What is $n$?

A  $n = 13$ because we are choosing from the 13 cards of the suit. There are $13C_2 = 78$ ways to pick 2 cards of the suit.
We’ve seen that there are 4 choices for the suit and 78 choices for picking the 2 cards of the suit.

We’ve also seen that there are $\binom{52}{2} = 1,326$ ways to pick 2 cards from the deck.

The total number of ways of getting 2 of the same suit is then $4 \cdot 78 = 312$. Then the probability of getting 2 of the same suit is $\frac{312}{1326}$, or a little worse than 1 out of 4 hands.
Suppose you have a pocket pair. What is the probability that you’ll get 3 of a kind on the flop?

Let’s say we have 2 Aces. In order to get 3 of a kind on the flop (the first three cards dealt up), we need to have one of the three cards be an Ace and the other two anything else. We need to select an Ace and select two other cards.

There are \(2 \binom{1}{1} = 2\) ways to select one of the remaining Aces.

There are \(48 \binom{2}{2}\) ways to select two non-Aces. This number is equal to \((48 \cdot 47)/2 = 1128\).
There are then $2 \cdot 1128 = 2256$ ways for this to happen. There are $\dbinom{50}{3} = 19,600$ total ways to select 3 cards.

The probability of hitting 3 of a kind on the flop is then $\frac{2256}{19600}$, which is about 1 time out of 9, or a little more than 10% of the time.

This does include getting 4 of a kind on the flop, but that happens rarely, so the probability of getting exactly 3 of a kind is almost what we just calculated.
Hitting 3 of a Kind with a Pocket Pair

If you have a pocket pair, what is the probability of hitting 3 of a kind by the end of the hand?

We’ve seen that the number of ways to deal the five face-up cards is \( \binom{50}{5} = 2,118,760 \).

If we have, say, a pair of Aces, to end up with 3 of a kind, of the five dealt cards one must be an Ace and the other 4 anything else.

To get this we must choose an Ace and choose 4 other cards. There are \( \binom{2}{1} = 2 \) ways to pick the Ace.
Q  How many ways are there to pick the 4 other cards? It is \( n \binom{4}{4} \) for some value of \( n \). What is the value of \( n \)?

A  \( n = 48 \). There are \( 48 \binom{4}{4} = 194,580 \) ways to pick the other cards.

Then \( 2 \cdot 194580 = 389160 \) total ways for this to happen.

The probability of hitting 3 of a kind is then \( 389,160/2,118,760 \), which is about 1 in 5.
Suppose you are dealt two spades. What is the probability you’ll end up with a flush?

In order to hit a flush, we must have 3 spades dealt along with 2 other cards.

There are \(_{11}C_3 = 165\) ways to pick 3 of the 11 spades remaining (2 are in our hand).

There are \(_{47}C_2 = 1081\) ways to pick 2 more cards. We can pick more spades and still end up with a flush.
The total number of ways the 5 cards can be dealt and we end up with a flush is then $165 \cdot 1081 = 178,365$.

The total number of ways the 5 cards can be dealt is $\binom{50}{5} = 2,118,760$.

The probability of hitting a flush is then $\frac{178,365}{2,118,760}$, or about 1 in every 12 hands that we are dealt two spades.
Suppose after the flop, you have 3 spades. What is the probability you’ll get a flush?

You need to get at two spade on the last two cards. At this point you see 5 cards, the 2 in your hand and the three on the board. So, the number of ways of dealing the final two cards is $\binom{47}{2} = 1081$.

In order to get two spades on the last two cards, you need to pick two of the 10 spades. Why 10? You are seeing 3, so there are 10 remaining to select. Then the number of ways of picking 2 of them is $\binom{10}{2} = 45$. 
The probability of hitting a flush when you have 3 spades at the flop is then $\frac{45}{1081}$, which is about 1 in 24 hands, or about 4%. Not too often!

Trying to get a flush is a desirable thing, but this doesn’t happen often, so you probably don’t want to chase flushes in this way.
Suppose after the flop you have 4 of 5 cards needed to make a straight, but you are missing a card in the middle. For example, you might have $4\spadesuit, 6\heartsuit, 7\spadesuit, 8\diamond$.

What is the probability of getting the straight? This is also called an inside straight.

There are $\binom{47}{2} = 1081$ ways to deal the last two cards. In order to hit your straight you must get a 5 and another card. There are 4 choices for the 5, and 46 choices for the remaining card.

There are $4 \cdot 46 = 182$ total ways to end up with your straight, so the probability is $\frac{182}{1081}$, or about 1 out of 6.
Suppose after the flop you have 4 of 5 cards needed to make a straight, and they are consecutive. For example, suppose you have $8\spadesuit, 9\heartsuit, 10\spadesuit, J\spadesuit$. What is the probability of hitting the straight?

Q Of the remaining cards in the deck, how many will give you a straight?

A 8. You must hit either a 7 or a Queen, and there are 4 of each.
There are 8 choices for the 7 or Queen, and 46 choices for the remaining card.

We have 46 because we see two in our hand and 4 on the board.

There are $8 \cdot 46 = 364$ ways to hit your straight. Since there are 1081 total ways to deal the last two cards, so your probability is $364/1081$, or 1 in 3. This is twice as likely as hitting an inside straight.
Next Week

We will finish up our discussion about Hold’em. We’ll also see a formula for expected value. Then we will discuss the odds of winning various New Mexico Lottery games. We’ll use the notion of expected value quite a bit to understand how much money the state can expect to receive from people playing the lottery. We’ll also see other applications of expected value.

We’ll also talk a little about odds versus probability. These are closely related but are not exactly the same.